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NUMERICAL STUDY OF THE NON-PERIODIC RESPONSE OF THE FORCED DUFFING'S OSCILLATOR

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COMMITTEE DECISION

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DEDICATION

To my parents.

ACKNOWLEDGEMENT

I would like to express my deepest thanks to Dr. Mazen Qaisi for his valuable advice throughout all stages of this work.

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NOMENCLATURE

F	The amplitude of the exciting force.
f, f_1, f_2, f_3, f_4, g, g_1, g_2, g_3, and g_4	Arbitrary functions used for the problem formulation in the Runge-Kutta method.
h or Δt	Time step.
M	Number of periods corresponding to the exciting force.
T	Period of the exciting force.
t	Time.
v_0	Initial velocity.
x	Displacement
x_0	Initial displacement.
y	Velocity.

Greek Symbols:

α	The linear stiffness.
β	The nonlinear stiffness.
δ	The coefficient of viscous damping.
ω	The angular frequency of the exciting force.

Abbreviation:

AMP	The spectral amplitude.
D	The deviation of the vibration frequency.
FREQ	The spectral frequency.
MEAN	The mean value of the vibration frequency.
PD	The percentage deviation of the vibration frequency.
VF	The vibration frequency.

ABSTRACT

NUMERICAL STUDY OF THE NON-PERIODIC RESPONSE OF THE FORCED DUFFING'S OSCILLATOR

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In the present study the forced Duffing's equation which represents the equation of motion of the forced Duffing's oscillator was solved numerically using the fourth order Runge-Kutta method. The solution was studied with assistance of the time history and the phase plane trajectory plots in a range of different system parameters. Frequency spectrum plots were plotted out to know the frequency contents of the output waveforms at certain cases. In general the response is non-periodic and the periodic response is a special case of it.

The type of motion resulting for different parameters of the system was classified into periodic, and non-periodic responses. The vibration frequency was found and the percentage deviation from the mean value was calculated. The effect of varying the parameters of the system on the type of motion as well as on the vibration frequency of the system was studied.

It was found that in the case of the periodic response, the waveform repeats itself at equal intervals of time, the corresponding phase plane trajectory plots are fine closed orbits. For the non-periodic response the repetition of the waveform is not constant but there is a small amount of shifting, the corresponding phase plane trajectory plots are not fine closed orbits but instead they spread from the both sides or could take certain regular shapes. Also it was found that as the amplitude of the exciting force increases, the amplitude and the frequency modulation effects increase, but these modulations disappear as the exciting frequency becomes much larger than the free vibration frequency.

It was shown that the vibration frequency obtained for periodic response was a horizontal straight line indicating a constant value, but for the non-periodic response it is fluctuating with time about a mean value and in general it is periodic. Also, It was shown that as the nonlinear stiffness increases, the vibration frequency increases. Increasing the amplitude and the frequency of the exciting force will also increase the vibration frequency. It was shown that the percentage deviation of the vibration frequency about its mean value increases with increases in the nonlinear stiffness and the forcing amplitude. Resonant conditions for the vibration frequency with maximum percentage deviation was found to occur when the exciting frequency is close to the free vibration frequency.

CHAPTER ONE

INTRODUCTION

1.1 Duffing's Equation & Its Importance:

The harmonically forced Duffing's equation is one of the most important nonlinear differential equations since it appears in various physical and engineering problems.

The forced Duffing's equation in a general form may be given as:

$$\ddot{x} + \delta\dot{x} + \alpha x + \beta x^3 = F\cos\omega t \qquad \text{..........(1.1)}$$

Subjected to the initial conditions:

$x(0) = x_0$ (initial displacement)

$\dot{x}(0) = v_0$ (initial velocity)

where

δ is the coefficient of viscous damping.

α is the linear stiffness.

β is the nonlinear stiffness.

F is the amplitude of the exciting force (harmonic excitation).

ω is exciting angular frequency.

and all derivatives are with respect to time t.

It should be noted that if the value of the linear stiffness parameter is positive, then the Duffing's oscillator is statically stable, if the value is negative , then the Duffing's oscillator is statically unstable, if the value equals to zero, then the oscillator is neutrally stable. Also, if the value of the nonlinear stiffness is positive, then the Duffing's oscillator is hardening, if the value is negative, then the Duffing's oscillator is softening. In this study the type of the oscillator is stable hardening oscillator.

Equation (1.1) is a second order nonlinear ordinary differential equation which has no exact solution . So that approximate analytical methods (such as perturbation and harmonic balance, etc.) or numerical methods may be used to solve the equation.

The forced Duffing's equation could describe different systems of various field of study such as mechanical, electrical, and chemical systems. It represents the vibration of a mass attached to a nonlinear spring subjected to harmonic excitation. Or, it may represent the large oscillation of a pendulum subjected to harmonic excitation. Also, such equation can be assumed to represent the forced vibration of a buckled beam in its fundamental mode. Also, it could represent large bending deflections of an electromagnetically driven steel beam held pinned to fixed supports.

1.2 Solution Description of Duffing's Equation:

The forced Duffing's equation represents the equation of motion of a second order spring-mass system with cubic nonlinearity subjected to harmonic excitation. The solution of the nonlinear equation represents the response of such system. In general, the response may be considered as non-periodic response into which the periodic response is a special case of it. In the case of the periodic response, the waveform is repeated periodically, but this is not so in the general case; the non-periodic response. The response whether it is periodic or non-periodic is very dependent on the system parameters as well as on the initial conditions.

Among the variety of available descriptors of the solution of the Duffing's equation, the following are of great significance and commonly used by investigators :

(1) Time histories.

(2) Phase plane trajectories.

(3) Power spectral densities.

(4) Poincare' maps.

Time history plot is self explanatory because it is simply the plot of displacement x(t) versus time t. The results come directly from the solution of the nonlinear equation.

The phase plane trajectory plot is the plot of velocity $\dot{x}(t)$ versus displacement x(t) on the phase plane corresponding to certain initial conditions.

Power spectral density plot is necessary when the frequency content of a given signal is to be known.

Finally, Poincare' map plot is not as straightforward as the others. Generally speaking, it is a phase plane plot that record data at certain instants of time. It is important in studying chaotic systems and strange attractors.

1.3 Literature Survey:

In 1918, Duffing introduce a nonlinear oscillator with a cubic stiffness term to describe the hardening spring effect observed in many mechanical problems. Since then this equation which was known with his name became one of the important examples in nonlinear oscillation texts and research articles.

Most of the analytical methods used to solve the equation are approximate methods. The general approach is as follows:

(1) Assume that the "small" nonlinearity is separable from the linear part of the equation.

(2) Neglect the nonlinear terms, as a first approximation, to obtain a general solution.

(3) Use the generating solution with the original equation to get the corrective terms due to the nonlinearity.

However, those methods are limited and only apply to periodic solutions only.

Hamdan and Burton [1] have used the harmonic balance method to study the steady state periodic responses and its stability for a softening Duffing's oscillator. They have used two harmonics rather than one in the approximate solution, and then they have observed that the qualitative nature of the harmonic balance solution changed.

In [2], a theoretical discussion for finding conditions that result in periodic solutions of Duffing's equation has been presented by Mehri and Ghorashi. This has been done based on Green function and Schauder's fixed point theorems. The importance of periodic solutions, from a practical point of view, lies in the fact that all other non-periodic solutions of this equation, which result from other initial conditions, if stable, converges to periodic solutions. Thus the periodic solution express the steady state behavior of the system under consideration.

In [3], different approximate analytical methods have been used by Dinca and Teodosiu to solve the harmonically forced Duffing's equation at different cases such as the undamped case, the case of viscous and coulomb damping , and other cases.

The operational adaptation of the Lindsted-Liapounoff method by using the table of Laplace transformation and the shorthand notation have been used by Pipes [4] to obtain the steady state solution of the forced Duffing's equation.

A modification of the conventional Duffing's equation in which the linear stiffness is negative gives what is called an unstable Duffing's equation. Such an equation describes the vibration of a buckled beam or plate when only one mode of vibration is considered under the action of a prescribed external force.

Moon [8] have studied the chaotic vibrations of a cantilevered beam buckled by magnetic forces. This was done by studying experimentally the forced non-periodic vibrations about the multiple equilibrium positions of the beam using poincare' plots in the phase plane.

In [9], the dynamics of a buckled beam have been studied for both the initial value problem and the forced external excitation by Dowell and Pezeshki. The principal focus was on chaotic oscillations due to forced excitation. They have made a comparison of results from a theoretical model with those from a physical experiment and they have shown that there was generally a good agreement between them.

Dowell in a separate paper [10], have also made experiments on the buckled beam. he has considered the relationship between chaos induced by forced oscillations versus self excited oscillations. He has seen that the initial value problem for a second order homogeneous system is a key to the understanding of higher order system, including the inhomogeneous second order system. he also has made a comparison between theoretical results for Duffing's equation and (physical) experiments for a buckled beam and this have shown generally good agreement.

Tang and Dowell [11] have investigated experimentally the chaotic behavior of a buckled beam for forced deterministic excitation. In their paper, they have included the effects of higher modes on the chaotic response. They have studied the influence of exciting force and damping on the system response. They have seen that there was generally good quantitative agreement between theoretical and experimental results obtained by including up to three beam modes.

In this study the fourth order Runge-Kutta method will be used to solve the forced Duffing's equation numerically. After solving, the vibration frequency will be found and its nature will be studied. Also, the effect of varying the parameters of the system on the type of motion as well as on the vibration frequency will be studied. By considering the works done in the literature we note that there are no works available in the literature dealing with the nature of the vibration frequency of the Duffing's system and the effects of varying parameters on it.

CHAPTER TWO

SOLUTION PROCEDURES

2.1 Problem formulation & Solution Procedures of the Duffing's Equation:

The fourth order Runge-kutta method was used to solve Equation (1.1) numerically. The second order differential equation is first reduced to two first order differential equations.

By letting $\dot{x} = y$, Equation (1.1) is reduced to the following two first order equations

$$\dot{x} = y \quad x(0) = x_0$$
$$\dot{y} = F\cos\omega t - \delta y - \alpha x - \beta x^3 \quad y(0) = v_0$$

..........(2.1)

So that, in general form

$$\dot{x} = f(t, x, y)$$
$$\dot{y} = g(t, x, y)$$

..........(2.2)

Where f and g are arbitrary functions.

The computational equations to be used are programmed for digital computer, the program was written in the programming language FORTRAN (see Appendix C, Computer Program # 1).

The equations are in the following order :

$$f_1 = f(t_i, x_i, y_i) = y_i$$
$$g_1 = g(t_i, x_i, y_i) = F\cos\omega t_i - \delta y_i - \alpha x_i - \beta x_i^3$$

$$f_2 = f(t_i + \frac{h}{2}, x_i + \frac{h}{2} f_1, y_i + \frac{h}{2} g_1) = y_i + \frac{h}{2} g_1$$
$$g_2 = g(t_i + \frac{h}{2}, x_i + \frac{h}{2} f_1, y_i + \frac{h}{2} g_1) = F\cos\omega(t_i + \frac{h}{2}) - \delta(y_i + \frac{h}{2} g_1)$$
$$- \alpha(x_i + \frac{h}{2} f_1) - \beta(x_i + \frac{h}{2} f_1)^3$$

$$f_3 = f(t_i + \frac{h}{2}, x_i + \frac{h}{2} f_2, y_i + \frac{h}{2} g_2) = y_i + \frac{h}{2} g_2$$
$$g_3 = g(t_i + \frac{h}{2}, x_i + \frac{h}{2} f_2, y_i + \frac{h}{2} g_2) = F\cos\omega(t_i + \frac{h}{2}) - \delta(y_i + \frac{h}{2} g_2)$$
$$- \alpha(x_i + \frac{h}{2} f_2) - \beta(x_i + \frac{h}{2} f_2)^3$$

$$f_4 = f(t_i + h, x_i + hf_3, y_i + hg_3) = y_i + hg_3$$
$$g_4 = g(t_i + h, x_i + hf_3, y_i + hg_3) = F\cos\omega(t_i + h) - \delta(y_i + hg_3)$$
$$- \alpha(x_i + hf_3) - \beta(x_i + hf_3)^3$$

..........(2.3)

Where h is the time step Δt.

From the previous equations the values of x and y are determined form the following recurrence equations

$$x_{i+1} = x_i + \frac{h}{6}(f_1 + 2f_2 + 2f_3 + f_4)$$

$$y_{i+1} = y_i + \frac{h}{6}(g_1 + 2g_2 + 2g_3 + g_4)$$

..........(2.4)

Thus with i=0, at $t_1=t_0+\Delta t$, x_1 and $y_1$are found by substituting the values of x_0,y_0 at t_0 (the initial conditions) into Equations (2.3) and the recurrence Equations (2.4) .

In this study, Equation (1.1) was numerically integrated on a digital computer by using the fourth order Runge-kutta method. The time step for the numerical simulation was chosen to be $\Delta t=T/400$ where $T=2\pi/\omega$ is the period of the exciting frequency ω. It was chosen like this because it gives good results, the error in the fourth order Runge-kutta method is of order h^5 and hence the error is very small. Smaller time step were tried with no significance change on the results. The time was started form zero up to a reasonable value. This value was different according to the exciting frequency ω. For ω=0.1, the time was ended at five periods of the exciting force, this was denoted on the figures as M=5. For ω=1, M=30 and for ω =10, M=150. These values was chosen in order to represent the best configuration of the response.

/For simplicity, the damping effect has been ignored (δ=0).All results are for simulations started from the initial condition values of one for the displacement ($x(0)=1$) and zero for the velocity ($\dot{x}(0)=0$), linear stiffness is fixed constant (α=1),nonlinear stiffness takes the values(β

=0.1,1, 2, and 10) , also the exciting frequency takes the values (ω=0.1,1, 5, and 10). For this combination of the values of nonlinear stiffness and the exciting frequency, the amplitude of the exciting force F was varied from 0 to 1 with step of 0.1 .

After specifying all the unknowns; the values of the system parameters and the initial conditions, we can apply the computer program to obtain the solution of the nonlinear differential equation using the fourth order Runge-kutta method. The computer program returns sequence of values corresponding to displacement and velocity at discrete equispaced points as a function of time. We can then plot out these results using time history and phase plane diagrams in assistance with a graphics software. these were shown on figures from Figure (3.1) to Figure (3.27).

2.2 Evaluation of the Vibration Frequency:

The main objective of present study is to find the vibration frequency (VF) of the system and examine its dependence on time and study the effects of the system parameters on it.

The vibration frequency was found by the following steps:

(1) Considering the time history plot of the response, the output waveforms is divided into periods, this was done by finding the intersections of the waveforms to the x-axis.

(2) Measuring the length of each period.

(3) Evaluating the frequency of each period by applying (VF=2π/T) where T is the length of the period, and also evaluating the time corresponding to each period which is the location of the center of the period.

(4) Plotting out the frequency of that periods versus their time to see how does the vibration frequency of the system changes with time.

To simplify the arithmetic operations, the previous procedures were programmed to deal with any expected shape of the output waveforms (see Appendix C, Computer Program # 2). Then, the results obtained from the second computer program were plotted out using a graphics software to obtain the vibration frequency (VF)-time plot. These were shown on the same figures used for the time histories and the phase plane trajectories; Figures (3.1) to (3.27).

2.3 Frequency Spectrum:

By using the fast Fourier transform (FFT) algorithm, the frequency spectrum; the plot of the spectral frequency versus the spectral amplitude of the output waveforms were obtained (see Appendix C, Computer Program # 3). These results plotted out and shown on Figures (3.32) and (3.33).

CHAPTER THREE

RESULTS & DISCUSSION

3.1 Results:

The results obtained from this study are presented at the end of this chapter. Figures (3.1) to (3.27) represent the solution of the nonlinear differential equation at certain values. For example, in Figure (3.1) the following values are taken: $\delta=0$, $\alpha=1$, $\beta=0.1$, $\omega=0.1$, $F=0$, $x_0=1$, $v_0=0$, and $M=5$. These values appeared on the first plot of Figure (3.1). For brevity, these values will appear after that in shortening case such as:

(0,1,0.1,0,1,0).
M=5

All the figures from Figure (3.1) to Figure (3.27) consist of three plots, the first one is the time history plot, the second is the phase plane trajectory plot, and the third plot is the vibration frequency(VF)-time plot. Under the third plot, the mean value of the vibration frequency was measured (which was denoted as MEAN) and presented there, also the deviation of the vibration frequency to the mean value was measured (which was denoted as D) and its percentage to the mean value was calculated (which was denoted as PD) and the results were presented there.

To study the effects of varying parameters of the system on the vibration frequency, certain figures were plotted out and these shown on Figures (3.28) to (3.31).

To know the frequency contents of the output waveforms, the frequency spectrum plots were plotted out; the plots of the spectral frequency (which was denoted as FREQ) versus the spectral amplitude (which was denoted as AMP). The results shown on Figures (3.32) and (3.33). Each figure consist of two plots, the first one is the frequency spectrum plot for the homogeneous Duffing's equation, and the second plot is the frequency spectrum plot for the forced Duffing's equation at forcing amplitude of one (F=1).

We should note that, not all the results were presented because they were very large, but instead a sufficient amount of them were presented which were useful in this study.

3.2 Discussion:

3.2.1 Choice of System Parameters & Initial Conditions:

For simplicity the damping effect was ignored for two reasons. Firstly, the nonlinearity is included in the stiffness term. Secondly, damping introduces additional effects which may obscure the dependence of the vibration frequency on the system parameters. The linear stiffness value was fixed, this value was chosen to be positive, so that we have a stable Duffing's oscillator. This oscillator has a single equilibrium position. The nonlinear stiffness was chosen to take positive values, thus we have a hardening Duffing's oscillator. These values were chosen to cover a wide range of cases. For the same previous reason, wide range of values for the exciting angular frequency were taken. The amplitude of the exciting force was varied from zero to one with step of 0.1 . The case when F=0 gives the homogeneous Duffing's equation. The initial displacement value was chosen to be one, the initial velocity value was chosen to be zero.

3.2.2 Classification of the Type of Motion:

As we said before, for zero force amplitude, we have a homogeneous Duffing's equation. The output response for this equation is periodic. For periodic response, the waveform repeats itself at equal intervals of time. The time history plot in Figures (3.1), (3.8), (3.12), (3.16), (3.20), and (3.24) represent a periodic response. In this case the phase plane trajectories corresponding to the previous figures in general form is an ellipse. It is a circle for Figures (3.1), (3.8), and (3.12) when the nonlinear stiffness is very small (β=0.1), this is because the system behaves as a linear system. The phase plane trajectory is an ellipse for Figures (3.16),(3.20), and (3.24) when the value of the nonlinear stiffness is not very small (β=1).

In all figures from the beginning up to Figure (3.27) except the figures associated with the homogeneous Duffing's equation described before, the response is non-periodic. In this case the waveform does not repeat itself regularly but there is a small amount of shifting which makes the response is not exactly periodic response. In the present study the time histories for the non-periodic response take certain shapes, such as beating waveforms, sinusoidal-like waveforms and others.

3.2.3 Effect of the Parameters of the System on the Output Waveforms:

Considering the first plot of Figure (3.1) (the time history plot), which shows the output waveforms for the homogeneous Duffing's equation where F=0. This situation represents the free vibration of the oscillator with its own characteristic properties. The value of the vibration frequency of the oscillator in this case appeared on the third plot of the same figure which is a constant value and it equals to 1.035 . We can observe also, there was an amplitude modulation effect on the output waveforms due to the increasing in the forcing amplitude from 0 to 0.1 . For the forced vibration case when F=0.1, ω=0.1 the output waveforms appeared on the first plot of Figure (3.2) makes us to believe that this output waveforms is obtained by superposing the free and the forced vibration cases, but we should keep in our mind that this is not a linear system but instead it is a nonlinear one, which means that there could be a relation between the input and the output but the relation is no longer linear as in the case for the linear systems. In that case the value of the vibration frequency decreased to 1.0299 due to the effect of the frequency modulation, because the forcing frequency was 0.1 and the free vibration frequency was 1.035 .

It is observed that forcing modulates the amplitude of the free vibration some times at the forcing frequency (0.1) and also modulates the free vibration frequency periodically at the forcing frequency. When the amplitude of the exciting force increases from 0.1 to 0.2, the effect of the forced vibration case increases as can be seen on the first plot (time history plot) of Figure (3.3), in this case the value of the vibration frequency decreased to 1.0245 as can be seen in the third plot of Figure (3.3) (vibration frequency-time plot). The same thing can be seen in Figures (3.4) to (3.6) where the increasing in the forcing amplitude increases the amplitude and frequency modulation effects. In Figure (3.7) the value of the vibration frequency was 0.10025, in this case the forced vibration case is dominant because the value of the vibration frequency was closed to the value of the forcing frequency. The same thing can be observed if we followed Figures (3.17) to (3.19).

When the frequency of the exciting force is close to that of the vibrating system, the output are beating waveforms. This can be observed for Figures (3.9) to (3.11) and Figures (3.21) to (3.23) where the forcing frequency is one (ω=1) and the frequency of the vibrating system equals 1.026 . In the first plot of Figure (3.9) (the time history plot) where F=0.1, the number of beating waveforms was two and a half, while it increased to six beating waveforms when the amplitude of the exciting force increased to 0.5 (F=0.5) as can be seen in the first plot of Figure (3.10). The number of the beating waveforms increased to nine beating waveforms when the amplitude of the exciting force increased to one (F=1) as can be seen in the first plot of Figure (3.11) which means that increasing in the forcing amplitude increases the amplitude modulation effect. The value of the vibration frequency for the homogeneous Duffing's equation in this case was 1.026 as can be seen in the third plot of Figure (3.8). When the forcing amplitude increases to 0.1(F=0.1), the value of the vibration frequency decreased to 0.9997 as can be seen in the third plot of Figure (3.9), then it decreased to 0.9775 when the forcing amplitude increases to 0.5 (F=0.5). This means that, increasing in the forcing amplitude increases the frequency modulation effect, because the effect of the forced vibration case increases which has a frequency less than the free vibration frequency. When F=1, the forced vibration case is the dominant case and thus the value of the vibration frequency increased to 0.98979 as can be seen in the third plot of Figure (3.11). The same thing observed for Figures (3.20) to (3.23), the value of the vibration frequency was 1.32 for the free vibration case, then it decreased gradually as the forcing amplitude increases until it reaches 0.99 at F=0.5, then it starts to increase due to the effect of increasing in the forcing amplitude, at which the forced vibration case is the dominant case.

In Figures (3.13) to (3.15) and Figures (3.25) to (3.27) where the forcing frequency equals ten (ω=10) and it is much greater than the free vibration frequency value, the system will vibrate in this case at a vibration frequency close to the free vibration frequency with some amplitude and frequency modulations.

Considering the second plot of Figure (3.1) (the phase plane trajectory plot) which is associated with the homogeneous Duffing's equation, we note from the plot that it is a fine closed orbit. The same thing was observed for Figure (3.8), (3.12), (3.16), (3.20), (3.24) into which the phase plane trajectory plot were fine closed orbits which represent a periodic response. As the forcing amplitude increases from 0 to 0.1, the trajectories spread from the both sides, as it can be seen from the second plot of Figure (3.2) (phase plane trajectory plot). This thickness increases as the amplitude of the exciting force increases as can be seen in Figures (3.3) to (3.7). The same thing was observed in Figures (3.13) to (3.15), Figure (3.9), Figures (3.17) to (3.19), Figure (3.21), and Figures (3.25) to (3.27).

In Figures (3.10) and (3.11), and Figures (3.22) and (3.23) the matter is different because the output waveforms are not simple waveforms (beating waveforms), even though we can observe the effect of increasing the forcing amplitude on the shape of the phase plane trajectory plots.

3.2.4 Vibration Frequency:

Examining the third plot of Figures (3.1) to (3.27) (the vibration frequency-time plots) we note that for the case of the homogeneous Duffing's equation the vibration frequency-time plot is a horizontal straight line indicating that its value is constant, this can be seen from the third plot of Figures (3.1),(3.8),(3.12),(3.16),(3.20), and (3.24).

By considering the third plot of Figure (3.2) (vibration frequency-time plot),we note that the shape of the vibration frequency looks like a sinusoidal curve. The same thing observed for Figures (3.3) and (3.4). In Figure (3.5) the top peaks became more flat than the lower peaks. In Figure (3.6) the top peaks take approximately the V-shape. In Figure (3.7) the vibration frequency-time plot takes the beating curve. For small forcing amplitude, the vibration frequency is modulated by the harmonic force at the forcing frequency (0.1).

In Figures (3.9) and (3.10) the shape of the vibration frequency-time plot looks like a sinusoidal curve. In Figure (3.11) the shape is generally periodic.

In Figures (3.13) to (3.15) and Figures (3.25) to (3.27) the vibration frequency-time plot take the beating curve while they were generally periodic for Figures (3.16) to (3.23).

We can conclude that in the case of the non-periodic response the vibration frequency-time plot is fluctuating. For these cases the mean value was measured and also the deviation and its percentage to the mean value were calculated. It is appeared from the plots that in general, the vibration frequency-time plot is periodic.

3.2.5 Effect of the Nonlinear Stiffness on the Vibration Frequency:

Figure (3.28) illustrates the influence of the nonlinear stiffness on the vibration frequency. The vibration frequency is evaluated at forcing amplitude of one (F=1), for two different values of exciting frequency ω =0.1 and 10 . The relation between them is a direct relationship, as the nonlinear stiffness increases, the vibration frequency increases as can be seen form the first and the second plots of Figure (3.28).

3.2.6 Effect of the Amplitude of the Exciting Force on the Vibration Frequency:

Considering Figure (3.29) which illustrates the influence of the forcing amplitude on the vibration frequency. The vibration frequency is evaluated at forcing frequency of ten (ω=10) because in this case the exciting frequency much greater than the free vibration frequency and thus the forced vibration case is the dominant case, it is evaluated for two different values of the nonlinear stiffness β=0.1 and 1, it is observed that the relation between the forcing amplitude and the vibration frequency is direct as can be seen from the two plots. For all values of the nonlinear stiffness as the amplitude of the exciting force increases, the vibration frequency increases.

3.2.7 Effect of the Frequency of the Exciting Force on the Vibration Frequency:

Figure (3.30) illustrates the effect of the exciting frequency on the vibration frequency. The vibration frequency was evaluated at a forcing amplitude of one (F=1), and at three different nonlinear stiffness values (β=0.1,1,2). It is observed that the relation between the vibration frequency and the exciting force is direct, as the exciting frequency increases, the vibration frequency increases for all values of the nonlinear stiffness.

The forgoing results show that the forcing frequency affects the mean vibration frequency to a far greater extent than either the forcing amplitude or the nonlinear stiffness.

3.2.8 Effect of the Nonlinear Stiffness on the Percentage Deviation of the Vibration Frequency:

Considering the first plot of Figure (3.31) which illustrates the relation between the nonlinear stiffness and the percentage deviation of the vibration frequency (PD). The values of the percentage deviation was calculated for ω=10, F=1. It is observed that the relation between the nonlinear stiffness and the percentage deviation almost constant for small nonlinear stiffness values and increase for large values of the nonlinear stiffness.

3.2.9 Effect of the Amplitude of the Exciting Force on the Percentage Deviation of the Vibration Frequency:

Considering the third plot of Figure (3.31) which illustrates the relation between the exciting amplitude and the percentage deviation of the vibration frequency . The values of the percentage deviation were calculated for β=1, ω=10. It is found from the plot that the relation between the forcing amplitude and the percentage deviation is direct, as the nonlinear stiffness increases, the percentage deviation increases too.

3.2.10 Effect of the Frequency of the Exciting Force on the Percentage Deviation of the Vibration Frequency:

The second plot of Figure (3.31) illustrate the influence of the exciting frequency on the percentage deviation of the vibration frequency. The values of the percentage deviation were calculated for $\beta=10$, $F=1$. The plot is a curve. At first, as the exciting frequency increases, the percentage deviation increases until it reaches a maximum value, then it begins to decrease. The maximum value was at forcing frequency equals to one which represents the resonant case for that oscillator into which the free vibration frequency and the forcing frequency are equal.

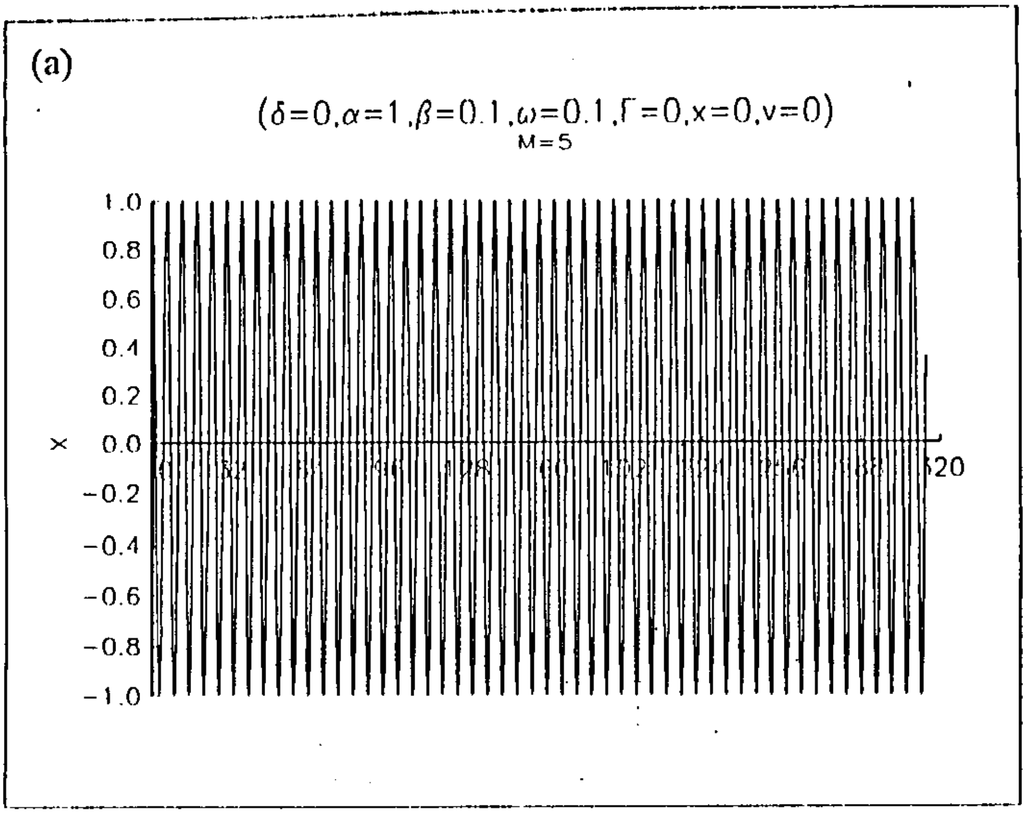
The results show that the forcing frequency affects the percentage deviation of the vibration frequency much more that either the forcing amplitude or the nonlinear stiffness. In addition, the vibration frequency is found to resonate when the forcing frequency equals the free vibration frequency.

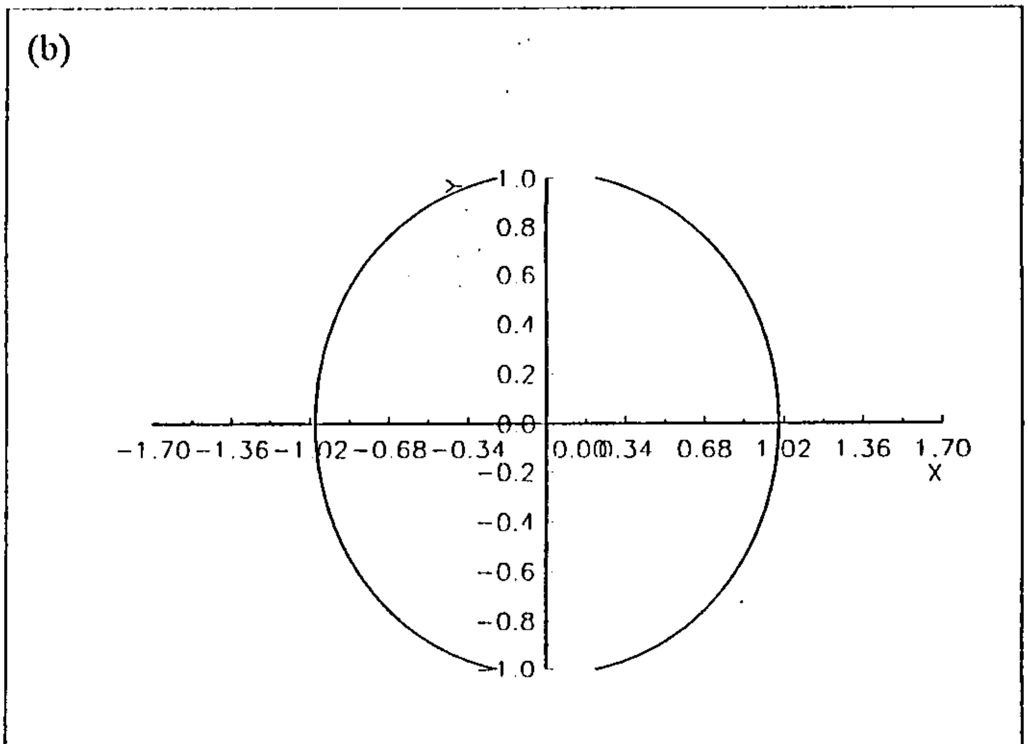
3.2.11 Frequency Spectrum:

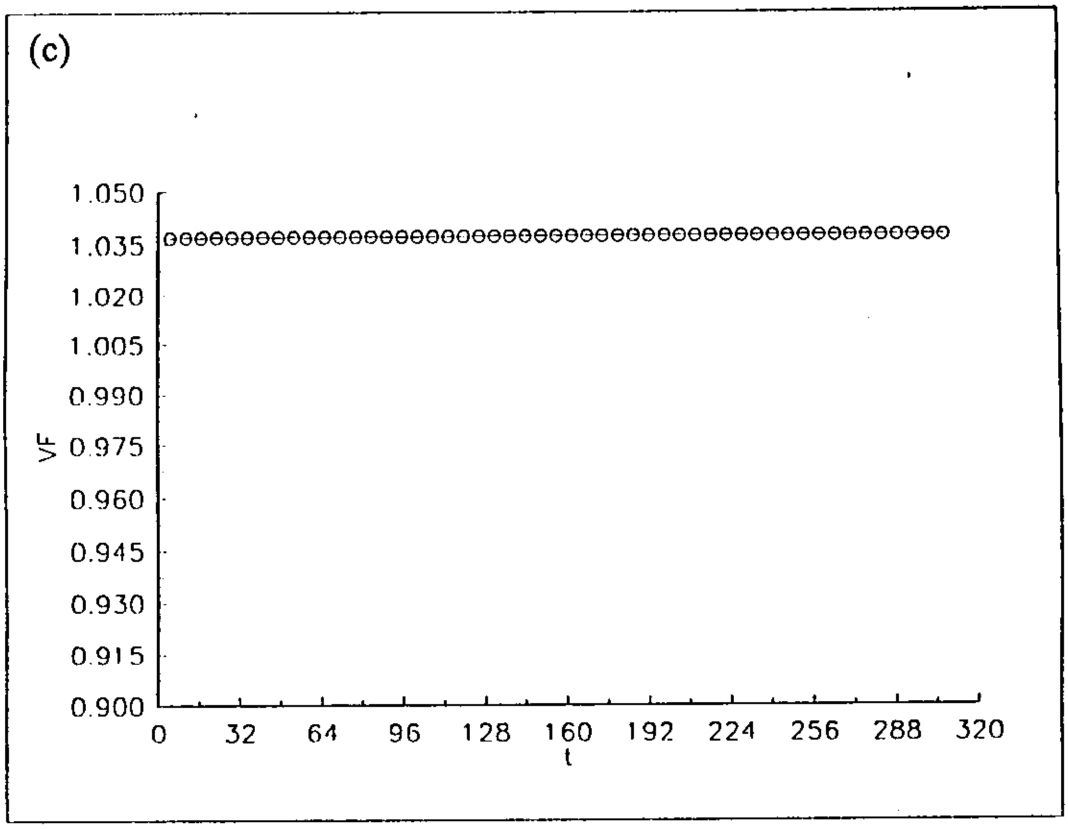
The frequency spectrum plots were found at $\beta=1$, and for values of forcing frequency $\omega=0.1$, and 1, these found for the homogeneous Duffing's equation and the forced Duffing's equation when the forcing amplitude equals one ($F=1$).

Considering the first plot of Figure (3.32) (the homogeneous Duffing's equation case) which shows that the spectrum consists of one frequency component which is the free vibration frequency at a value about 1.3 . In the second plot of the same figure where it was found at the forcing frequency of 0.1 (ω=0.1), there were a spectrum centered at the location of the free vibration frequency and other component for the forced vibration frequency at a value of 0.1 . We note also that the spectral amplitude decreased for the free vibration component in this case.

In Figure (3.33) the frequency spectrum plots were found at a forcing frequency of one (ω=1). In the second plot of this figure there were appeared a spectrum centered at the location of the free vibration frequency at a value about 1.3 and the other component for the forced vibration at a value of one. Also we note that the spectral amplitude decreased for the free vibration frequency component in this case.


(a)
(δ=0,α=1,β=0.1,ω=0.1,Γ=0,x=0,v=0)
M=5
1.0
0.8
0.6
0.4
0.2
x
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
520

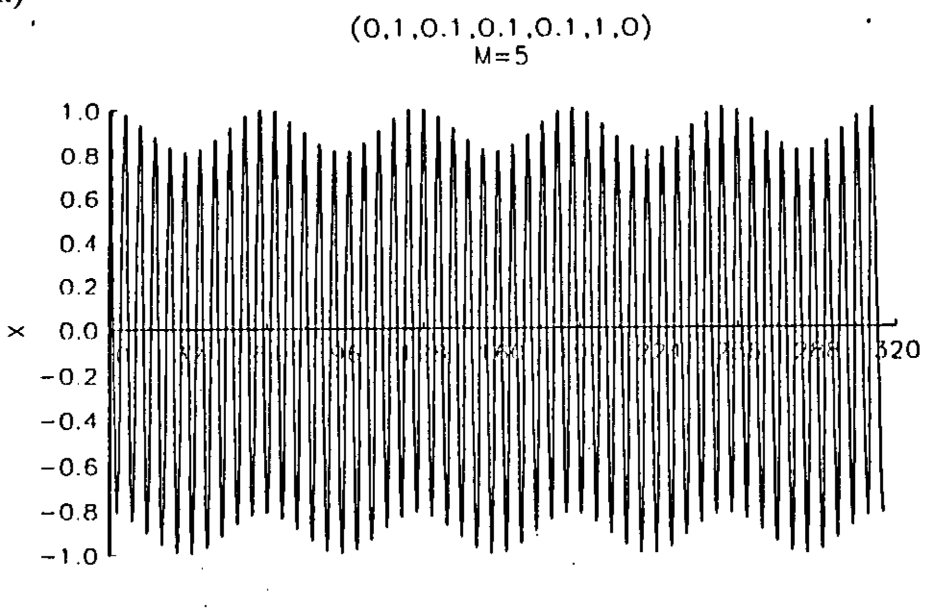

(b)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.70 -1.36 -1.02 -0.68 -0.34 0.00 0.34 0.68 1.02 1.36 1.70
X



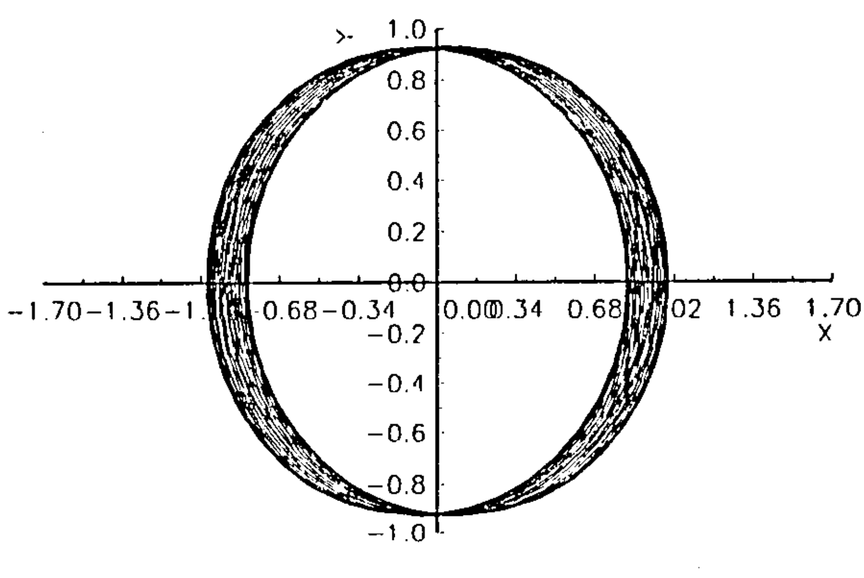
MEAN=1.035, D=0, PD=0

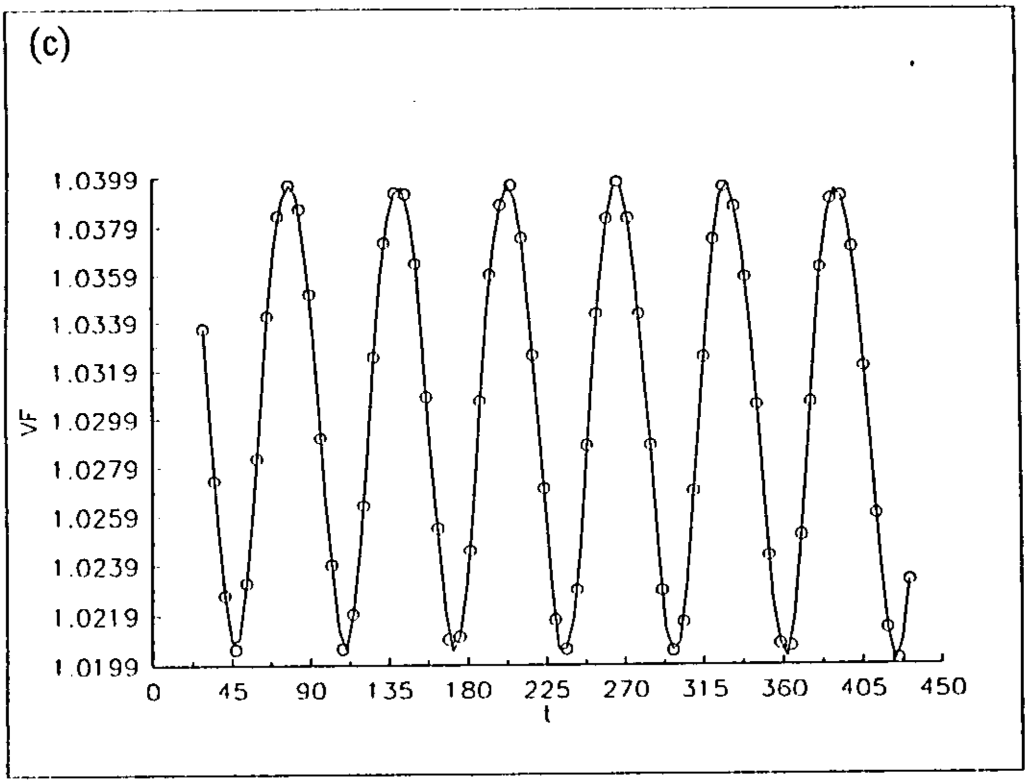
Figure (3.1) Time history, phase plane trajectory, and vibration frequency-time plots for β=0.1, ω=0.1, and F=0.

(a)


(0.1,0.1,0.1,0.1,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
x
320

(b)

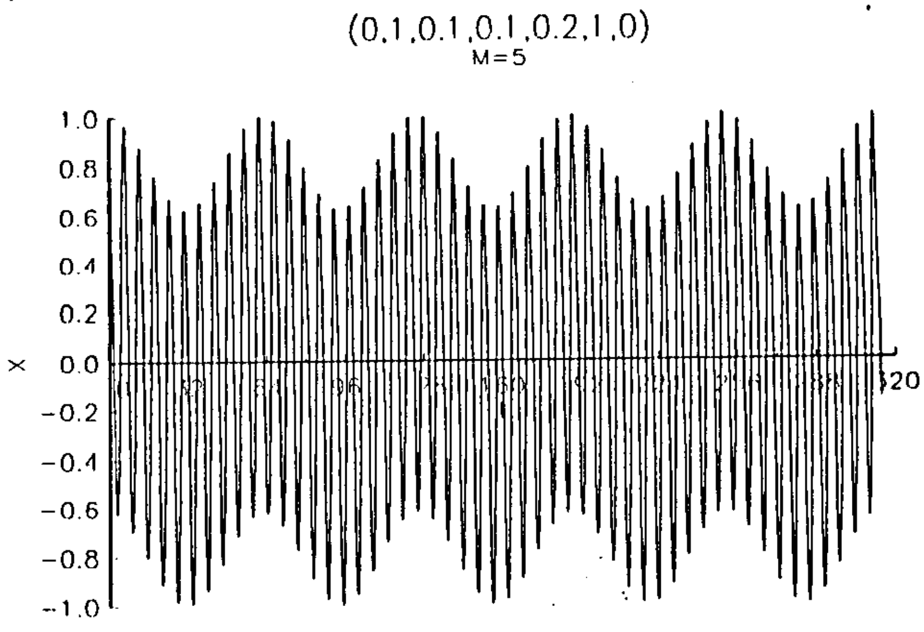

1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.70
-1.36
-0.68
-0.34
0.00
0.34
0.68
1.36
1.70
X



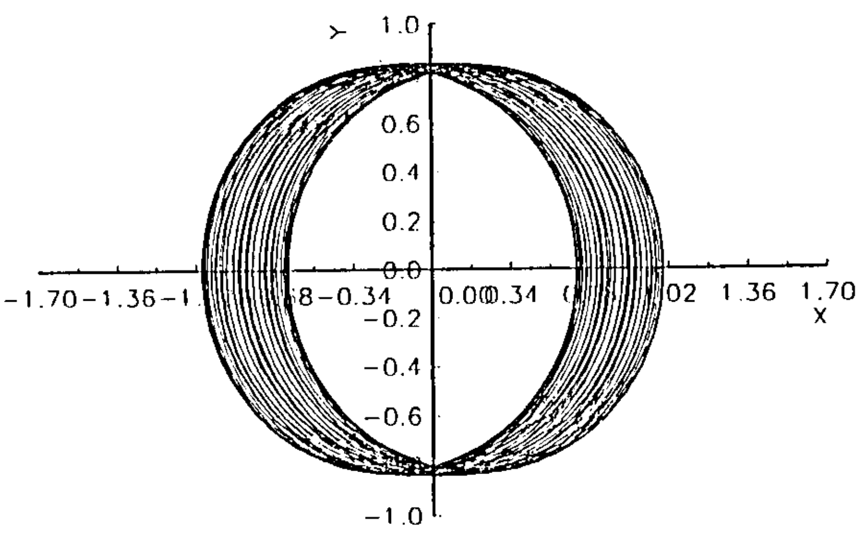
MEAN=1.0299, D=0.01, PD=0.97%

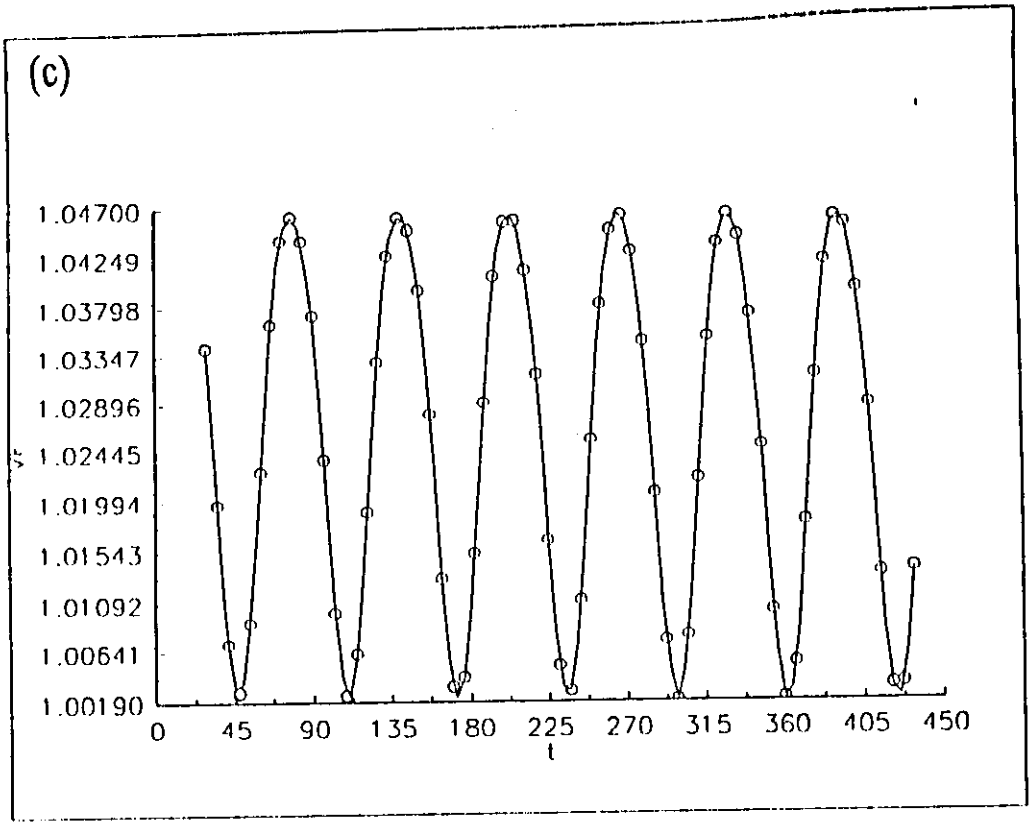
Figure (3.2) Time history, phase plane trajectory, and vibration frequency-time plots for β=0.1, ω=0.1, and F=0.1

(a)


(0.1,0.1,0.1,0.2,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
x
0.0
-0.2
-0.4
-0.6
-0.8
-1.0

(b)

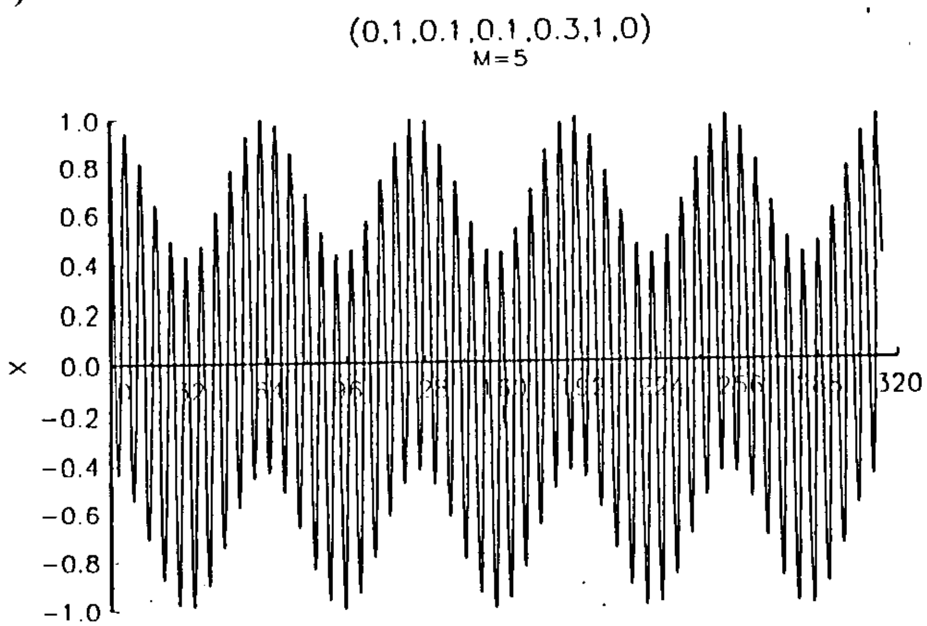

Y
1.0
0.6
0.4
0.2
0.0
-1.70
-1.36
-0.34
0.00
0.34
1.36
1.70
X
-0.2
-0.4
-0.6
-1.0



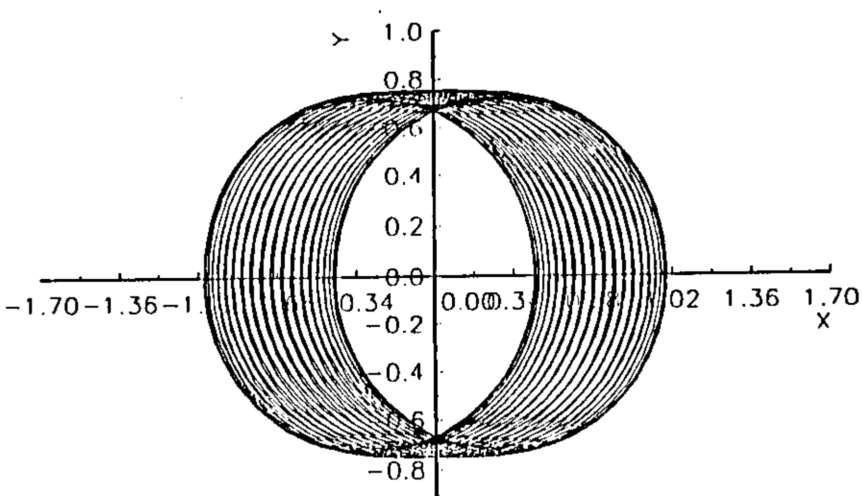
MEAN=1.0245, D=0.0226, PD=2.20%

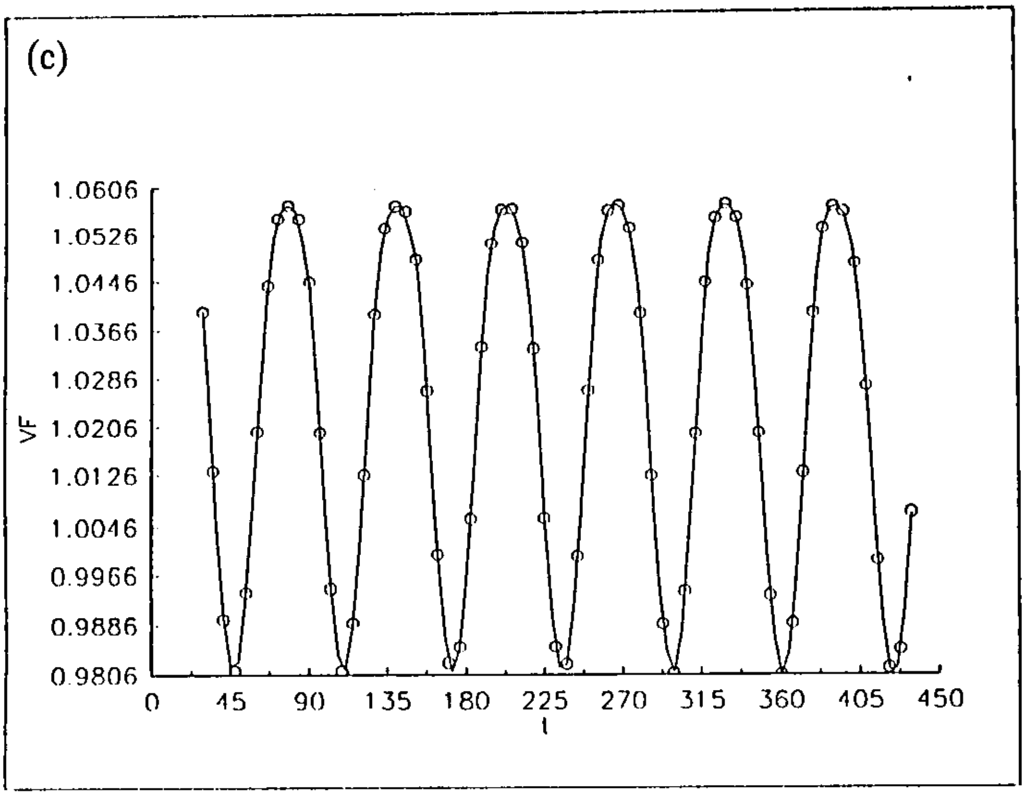
Figure (3.3) Time history, phase plane trajectory, and vibration frequency-time plots for $\beta=0.1$, $\omega=0.1$, and $F=0.2$

(a)


(0.1,0.1,0.1,0.3,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
x
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
320

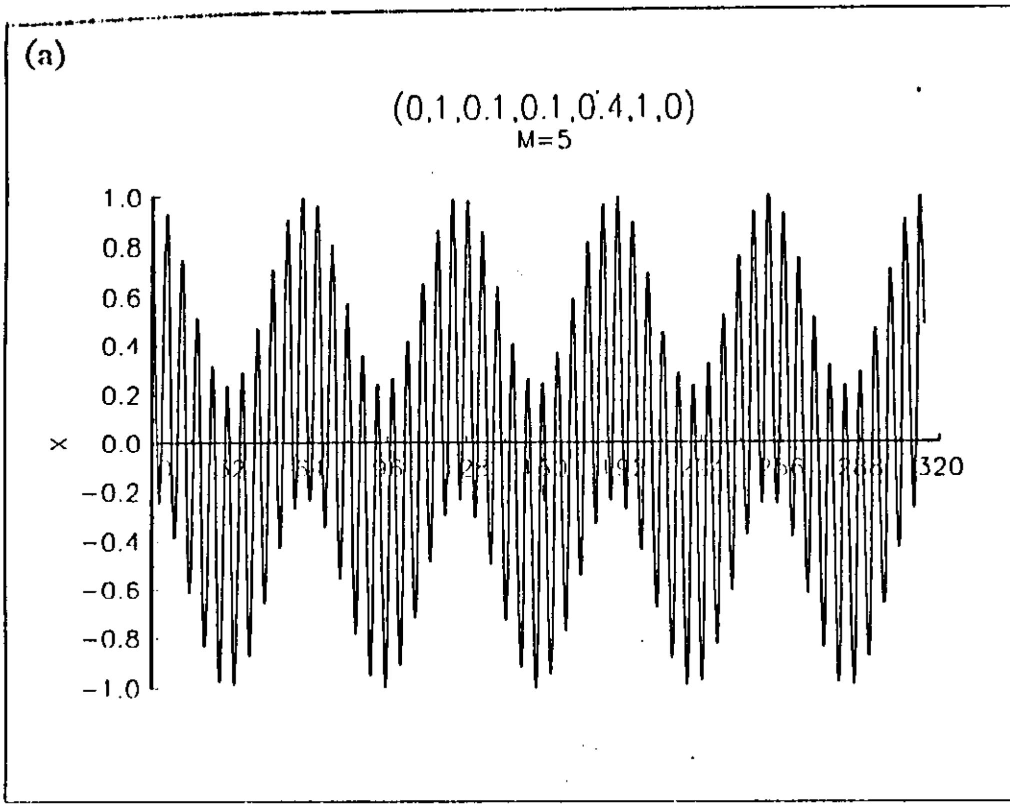
(b)

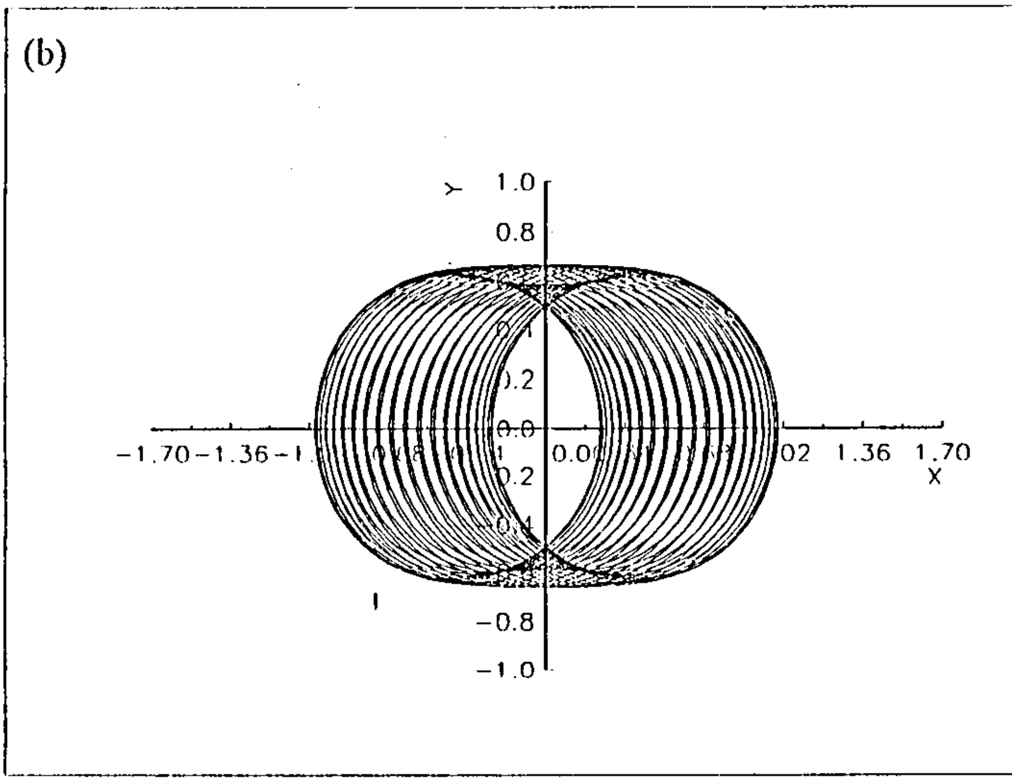

Y
1.0
0.8
0.4
0.2
0.0
-1.70
-1.36
0.34
0.00
1.36
1.70
X
-0.2
-0.4
-0.8
-1.0

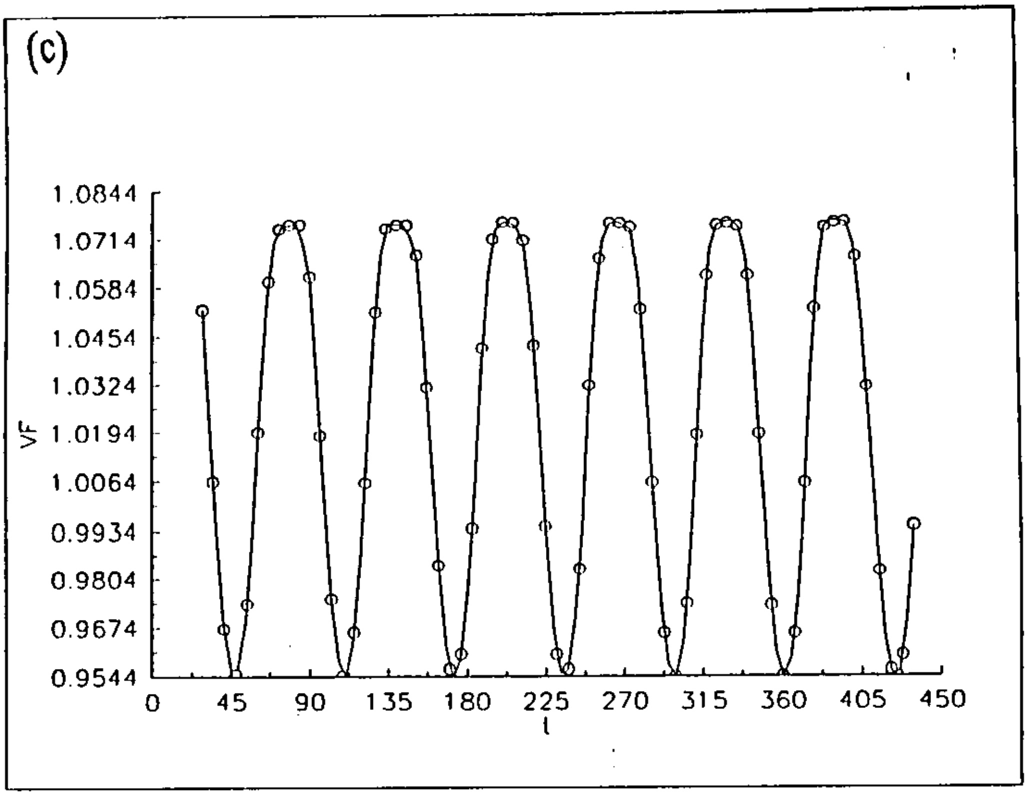


MEAN=1.0206, D=0.04, PD=3.91%

Figure (3.4) Time history, phase plane trajectory, and vibration frequency-time plots for β=0.1, ω=0.1, and F=0.3


(a)
(0,1,0.1,0.1,0.4,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
x
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
320

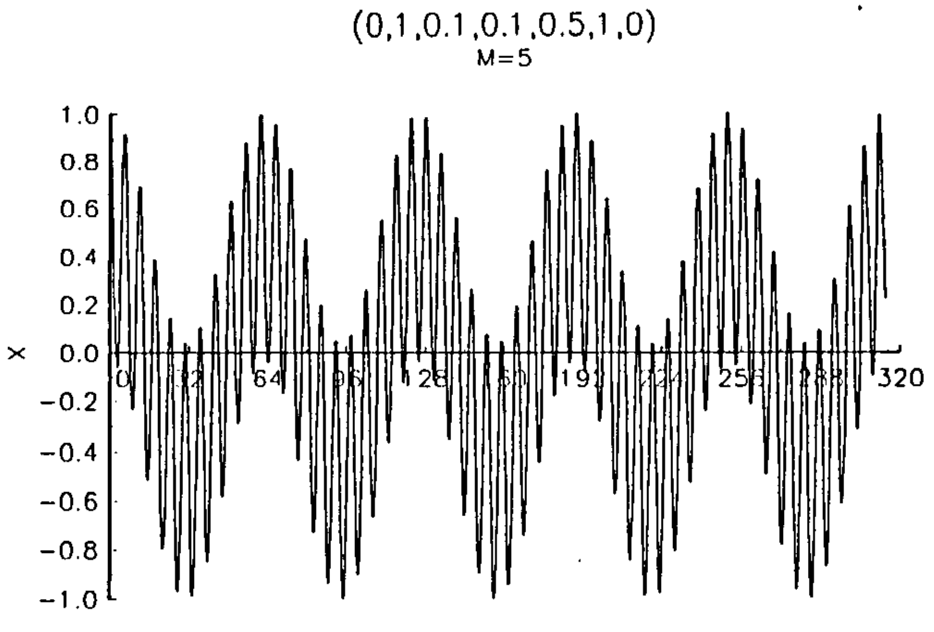

(b)
Y
1.0
0.8
-0.8
-1.0
-1.70
-1.36
1.36
1.70
X



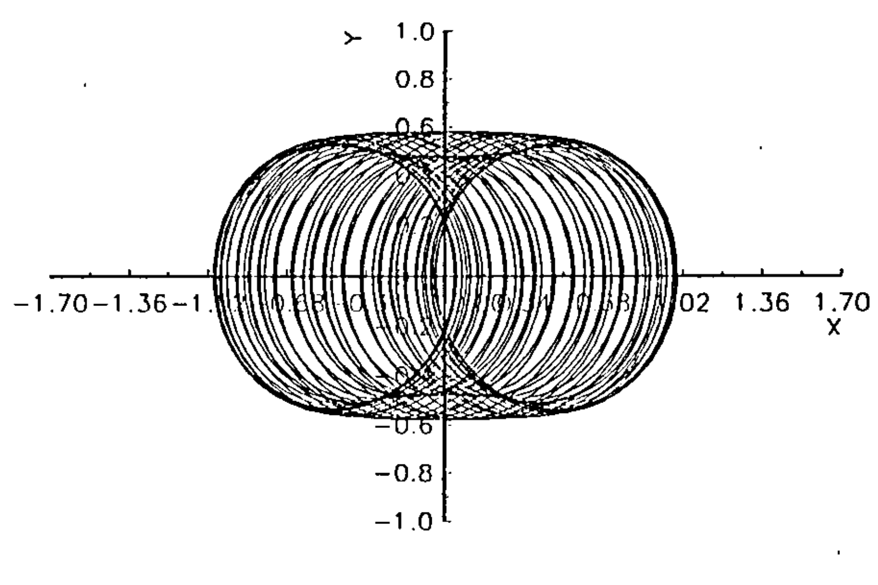
MEAN=1.01615, D=0.06175, PD=6.07%

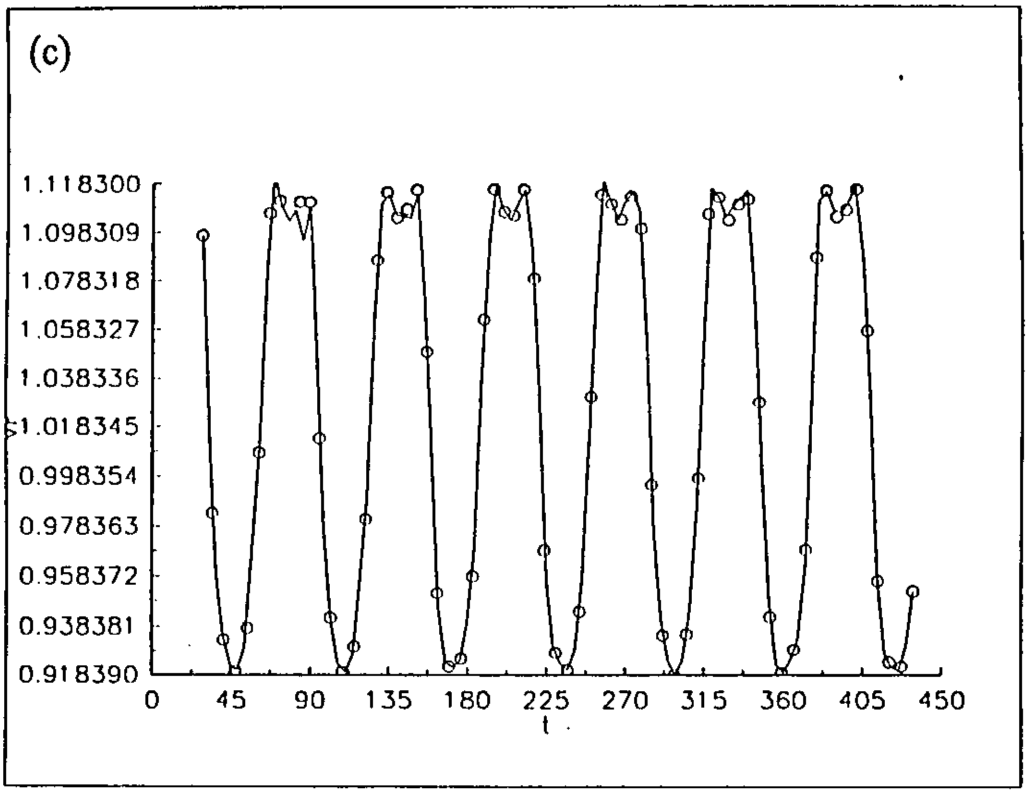
Figure (3.5) Time history, phase plane trajectory, and vibration frequency-time plots for β=0.1, ω=0.1, and F=0.4

(a)


(0,1,0.1,0.1,0.5,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
X
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
320

(b)

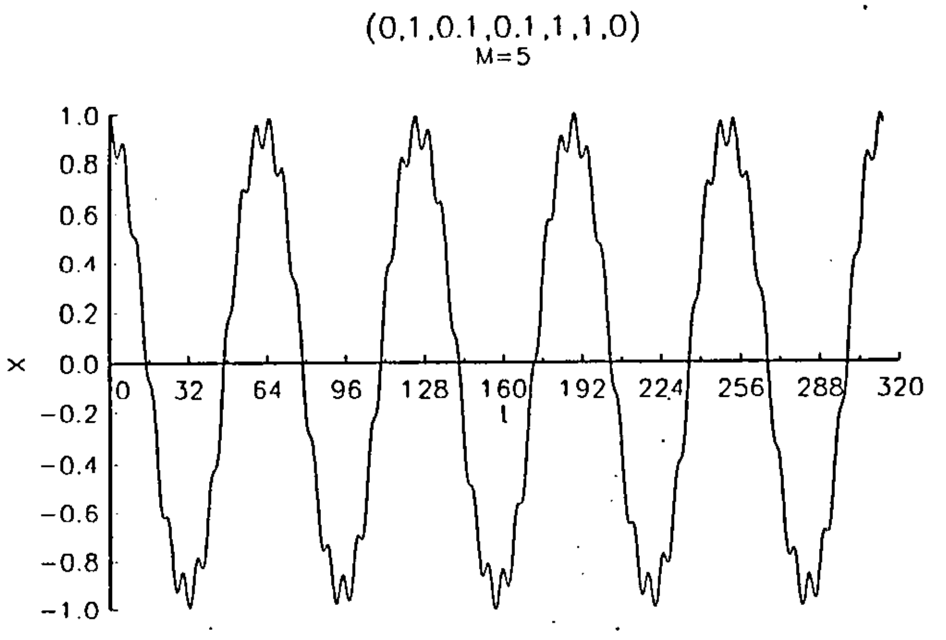

Y
1.0
0.8
-0.8
-1.0
-1.70 -1.36
1.36
1.70
X



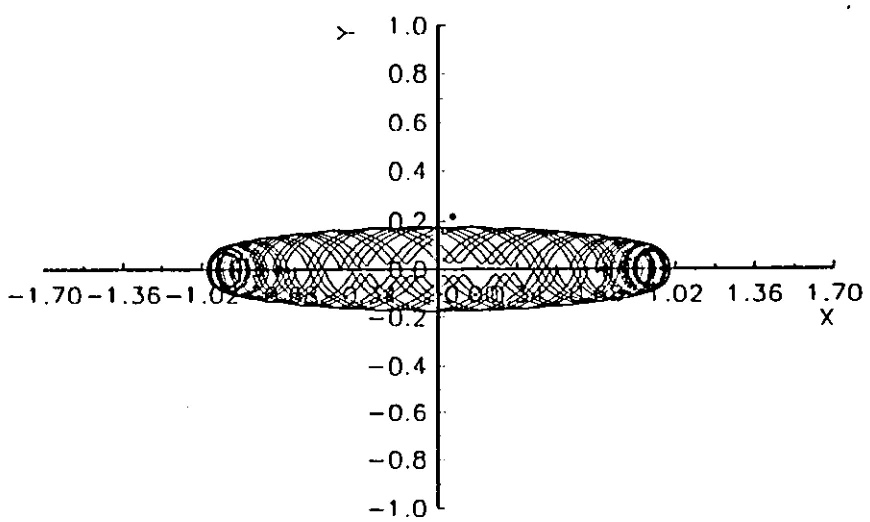
MEAN=1.01501, D=0.09562, PD=9.42%

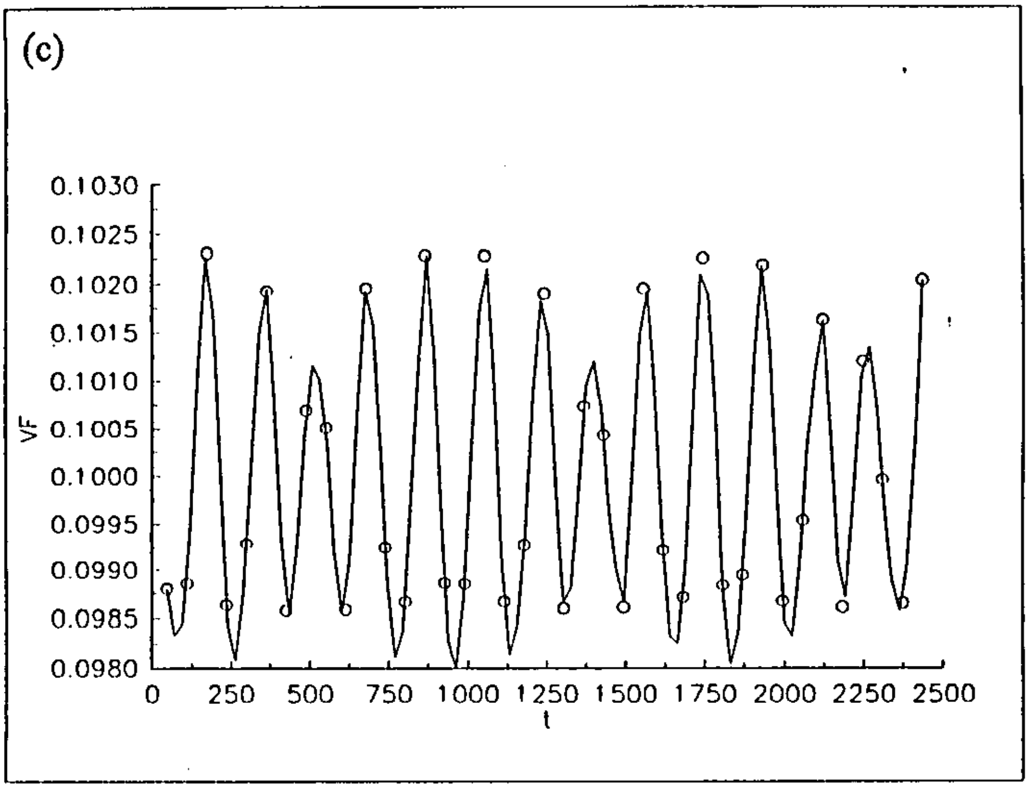
Figure (3.6) Time history, phase plane trajectory, and vibration frequency-time plots for β=0.1, ω=0.1, and F=0.5

(a)


(0,1,0.1,0.1,1,1,0)
M=5
x
t
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
32
64
96
128
160
192
224
256
288
320

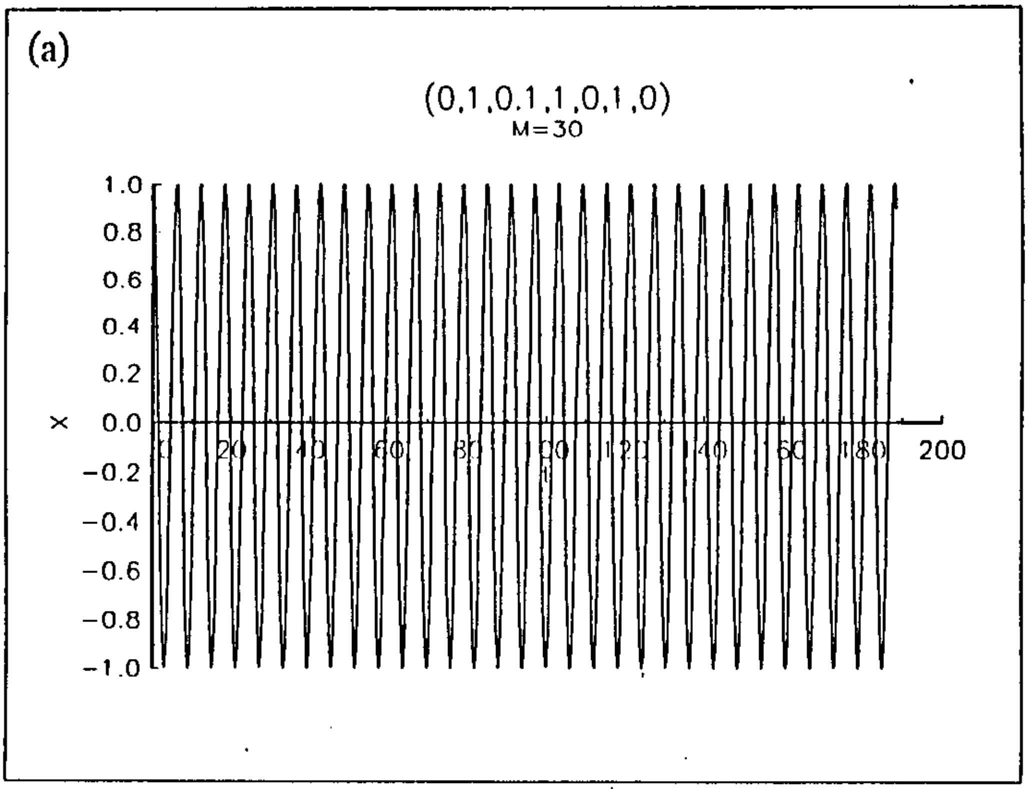
(b)

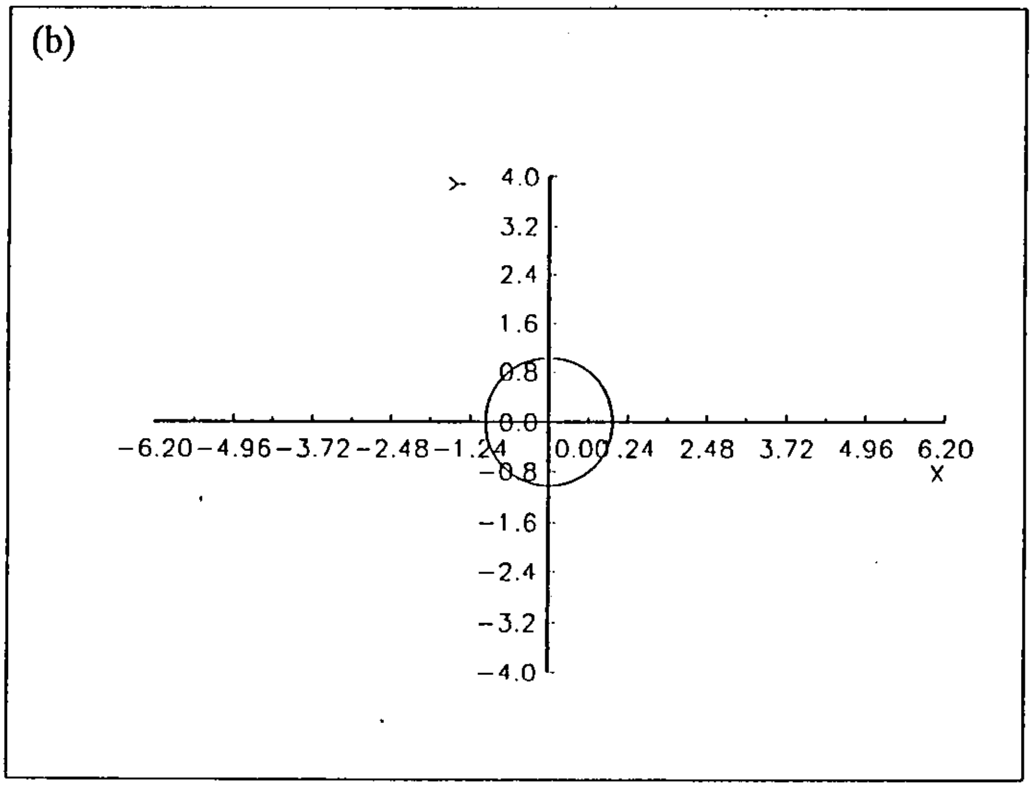

Y
X
1.0
0.8
0.6
0.4
0.2
−0.2
−0.4
−0.6
−0.8
−1.0
−1.70
−1.36
1.02
1.36
1.70

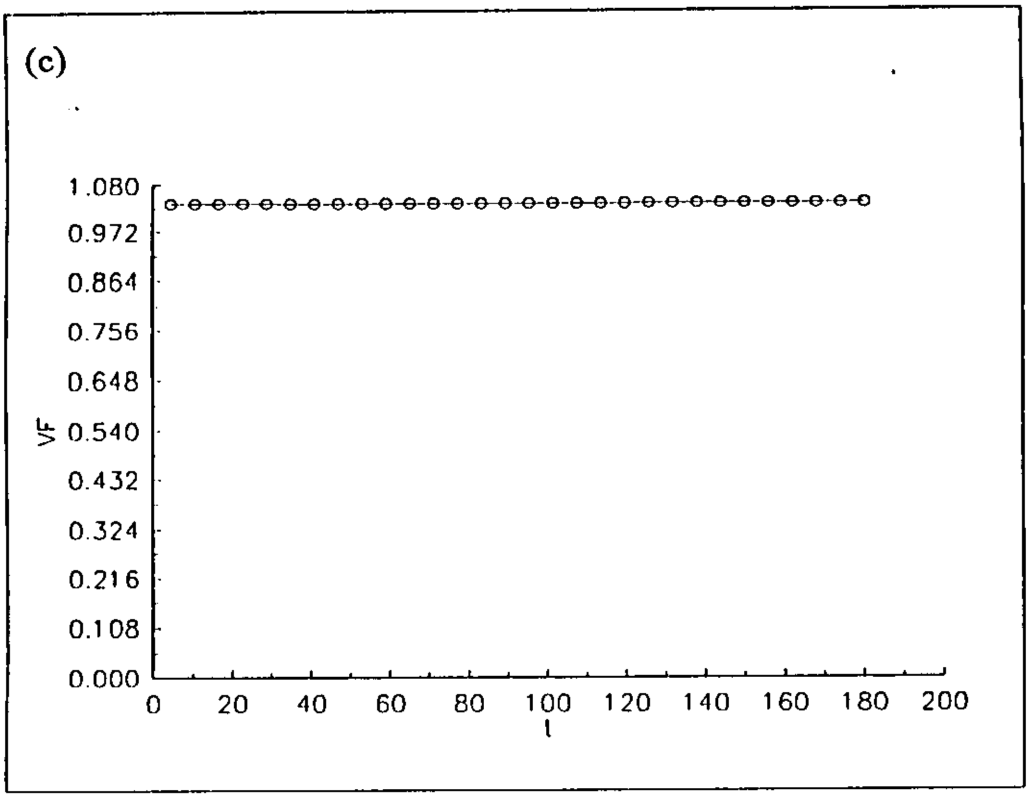


MEAN=0.10025, D=1.75 E-3, PD=1.74%

Figure (3.7) Time history, phase plane trajectory, and vibration frequency-time plots for β=0.1, ω=0.1, and F=1


(a)
(0,1,0,1,1,0,1,0)
M=30
1.0
0.8
0.6
0.4
0.2
x
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
200

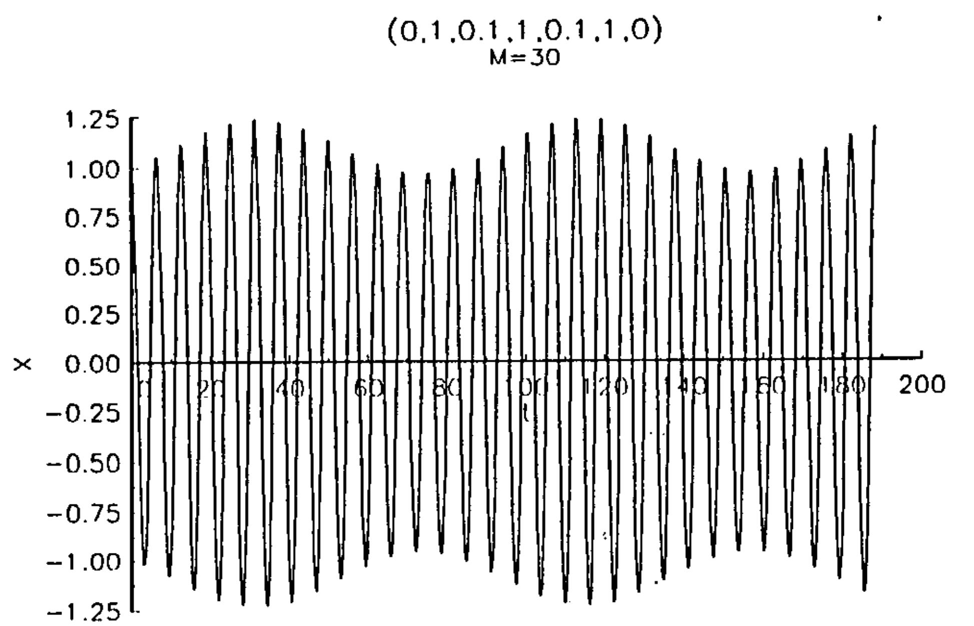

(b)
y
4.0
3.2
2.4
1.6
0.8
0.0
-0.8
-1.6
-2.4
-3.2
-4.0
-6.20
-4.96
-3.72
-2.48
-1.24
0.00
2.48
3.72
4.96
6.20
x



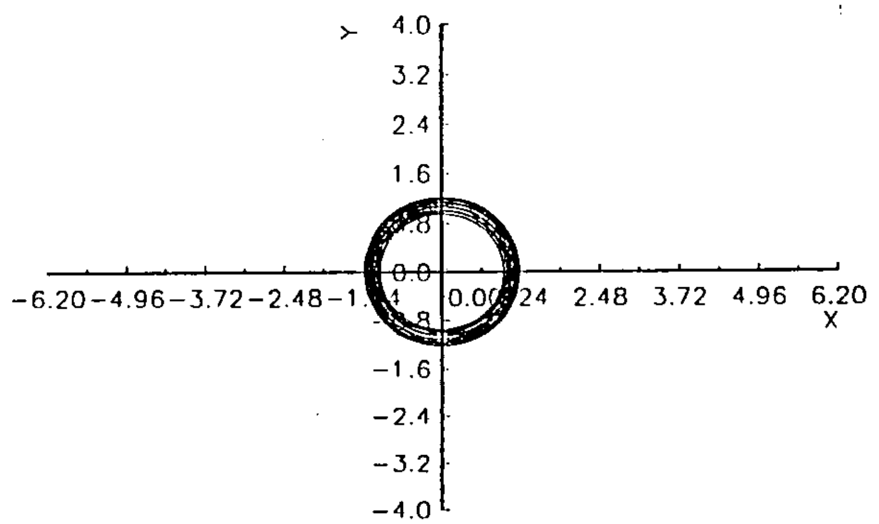
MEAN=1.026, D=0, PD=0

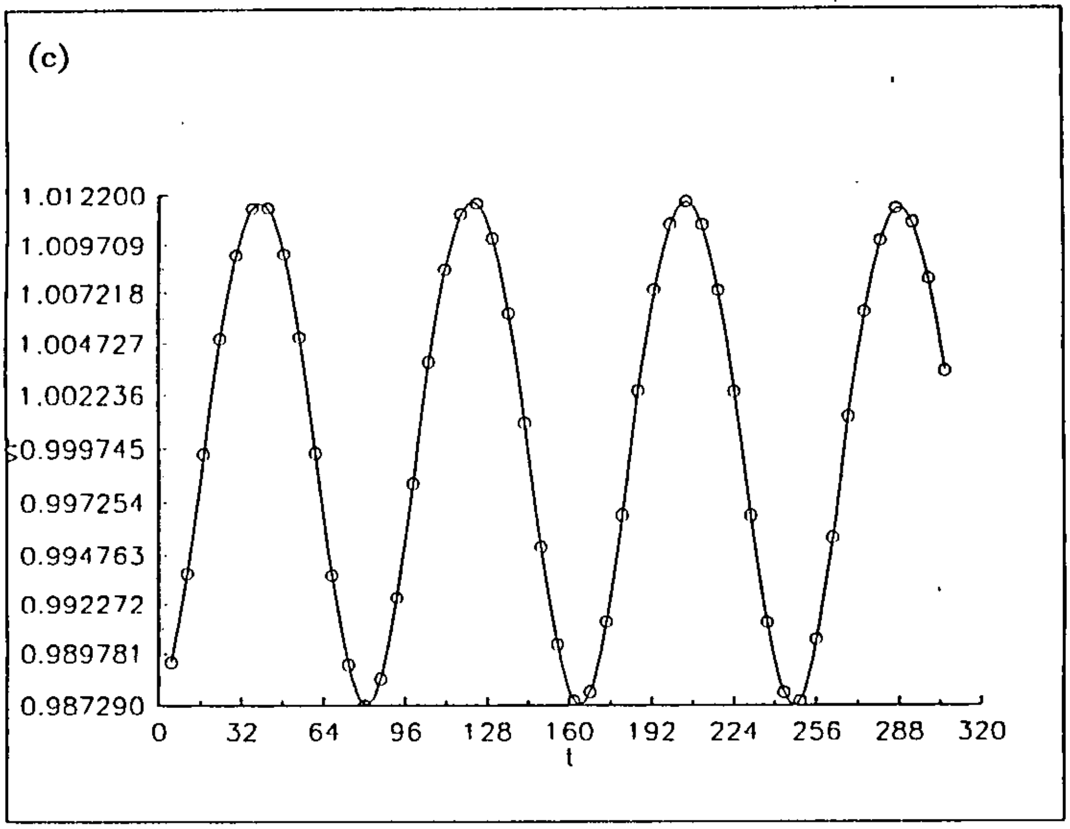
Figure (3.8) Time history, phase plane trajectory, and vibration frequency-time plot for $\beta=0.1$, $\omega=1$, and F=0.

(a)


(0.1,0.1,1,0.1,1,0)
M=30
1.25
1.00
0.75
0.50
0.25
x
0.00
-0.25
-0.50
-0.75
-1.00
-1.25
200

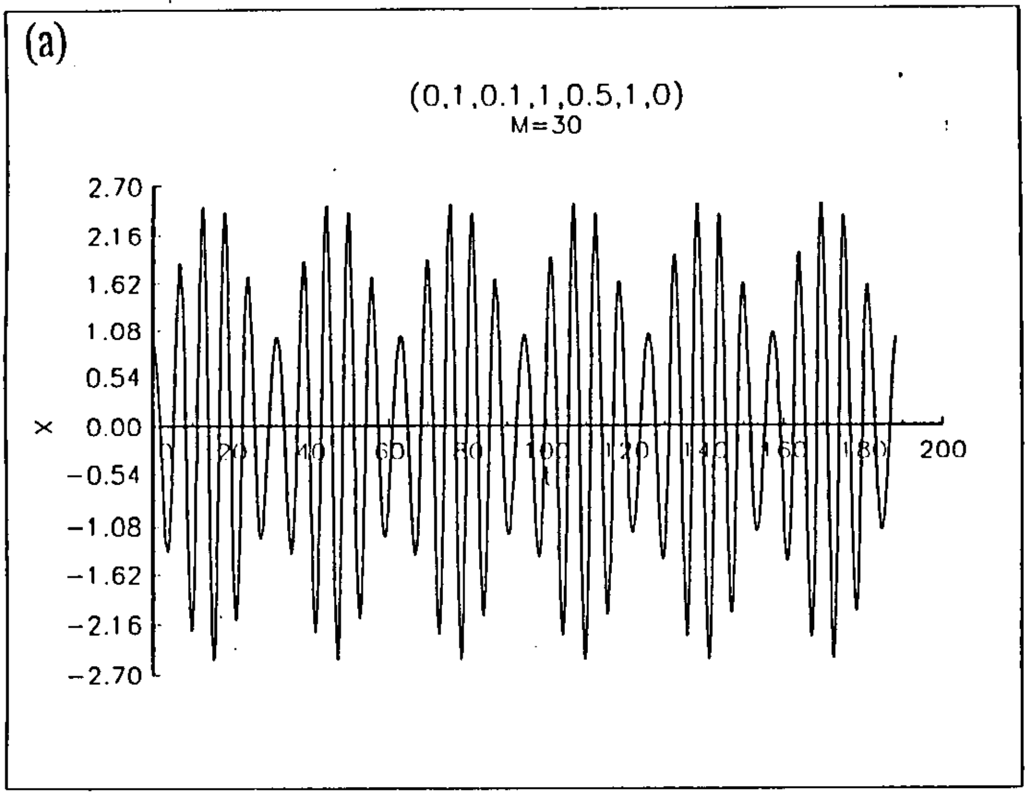
(b)

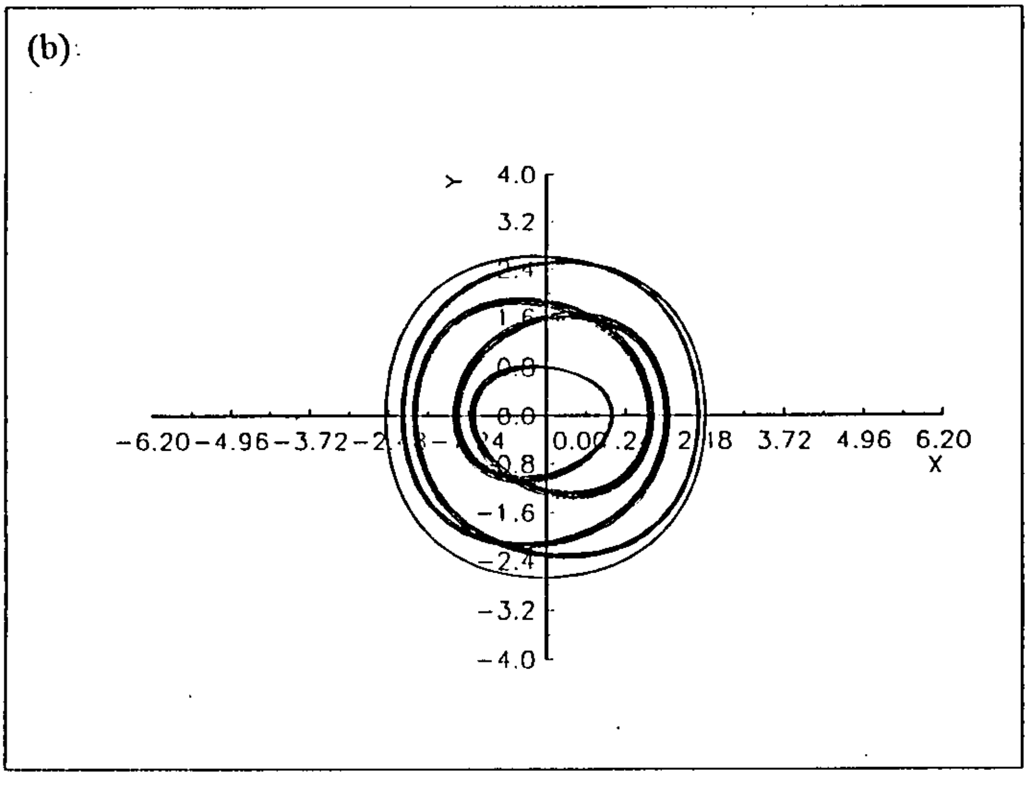

Y
4.0
3.2
2.4
1.6
-1.6
-2.4
-3.2
-4.0
-6.20 -4.96 -3.72 -2.48
2.48 3.72 4.96 6.20
X

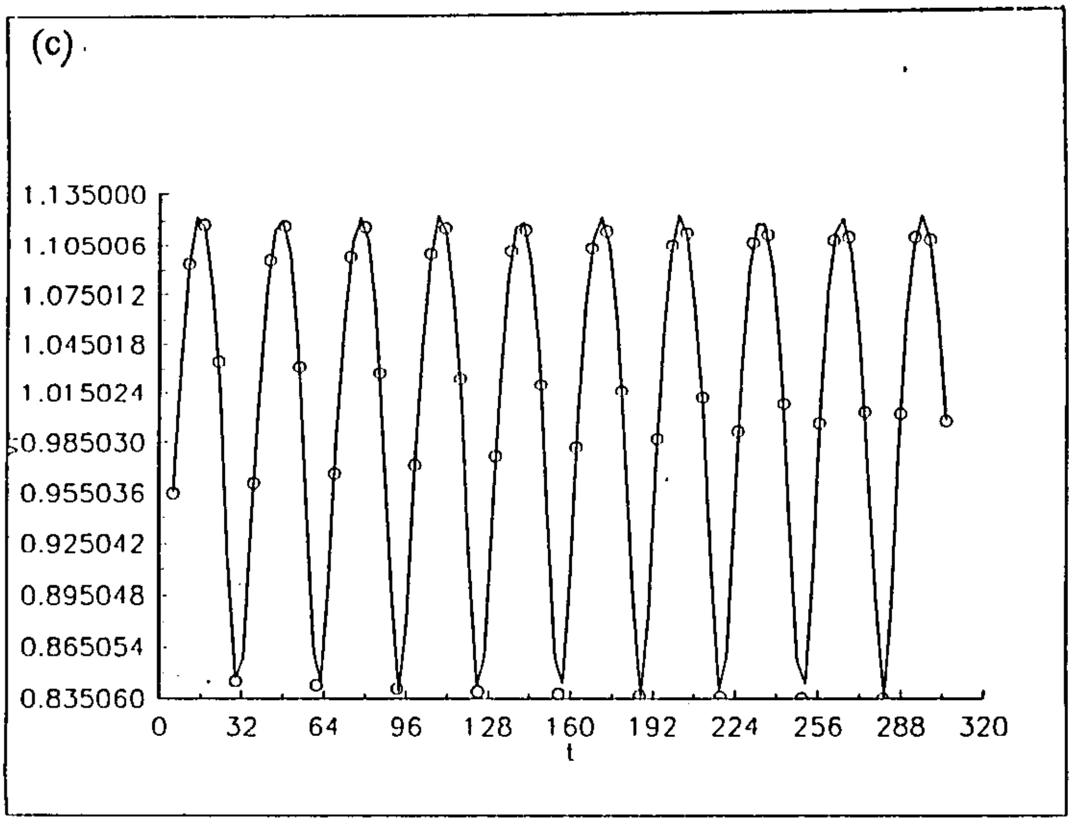


MEAN=0.9997, D=0.012455, PD=1.24%

Figure (3.9) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=1, and F=0.1

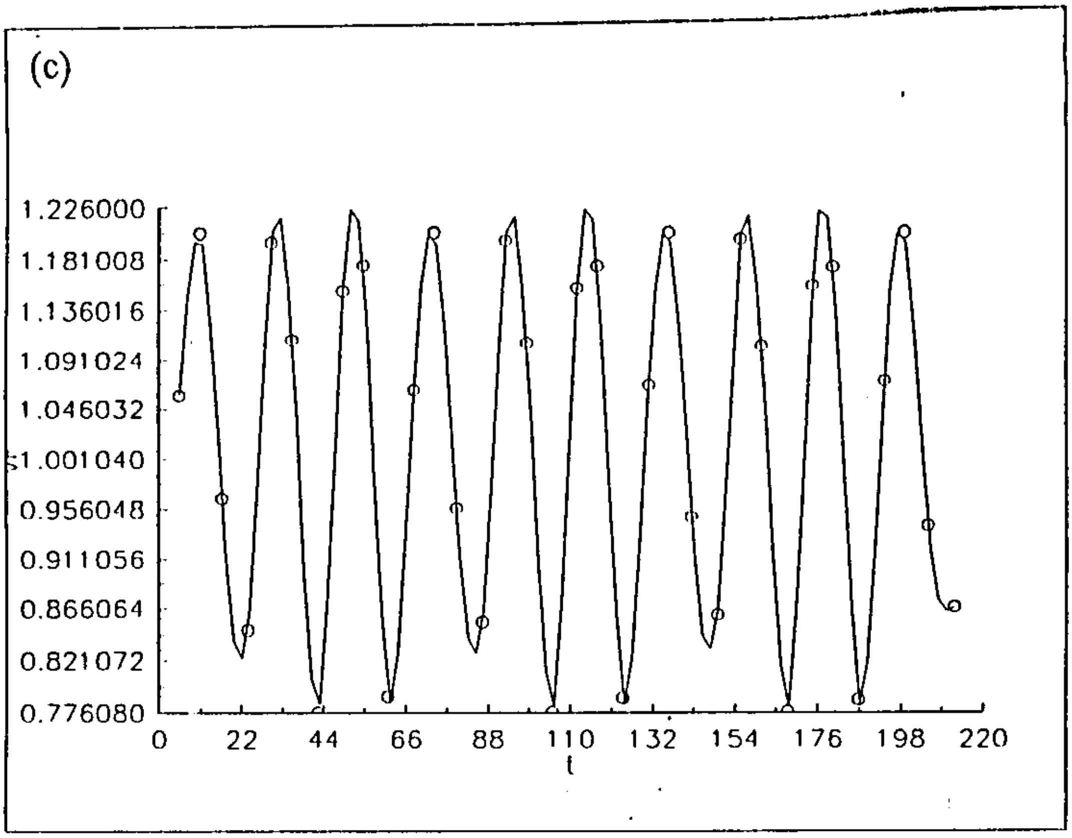

(a)
(0,1,0.1,1,0.5,1,0)
M=30
2.70
2.16
1.62
1.08
0.54
x
0.00
−0.54
−1.08
−1.62
−2.16
−2.70
200


(b)
Y
4.0
3.2
−1.6
−3.2
−4.0
−6.20
−4.96
−3.72
3.72
4.96
6.20
X



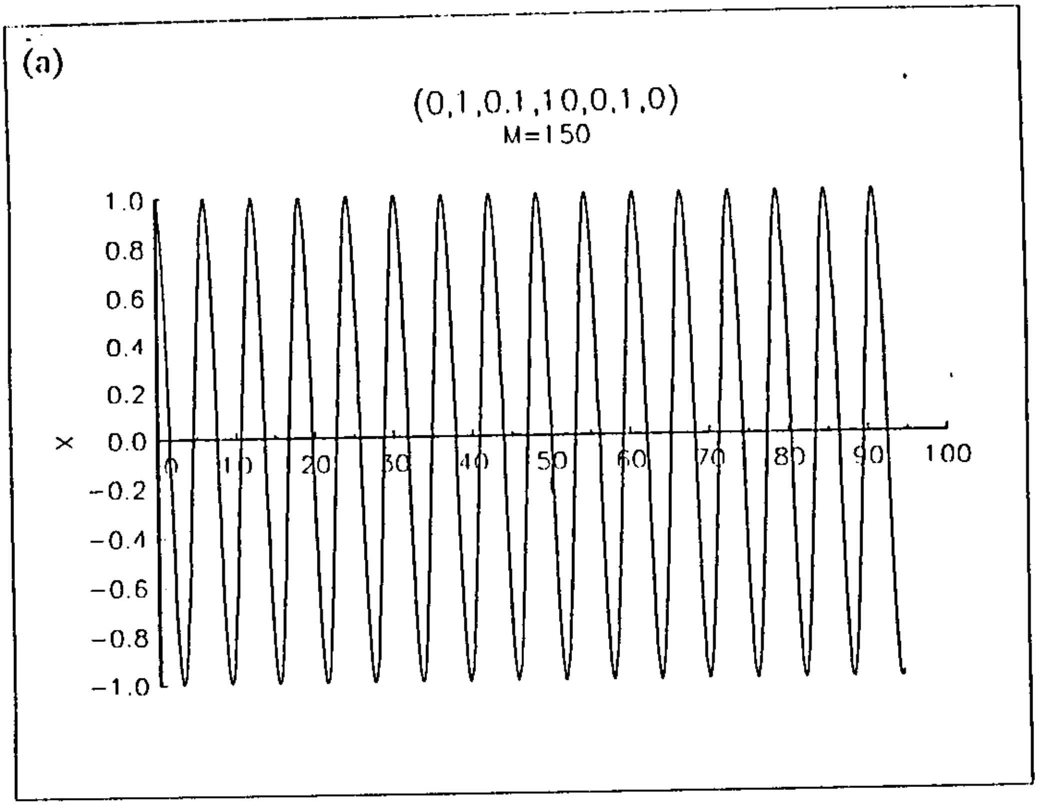
MEAN=0.9775, D=0.1425, PD=14.57%

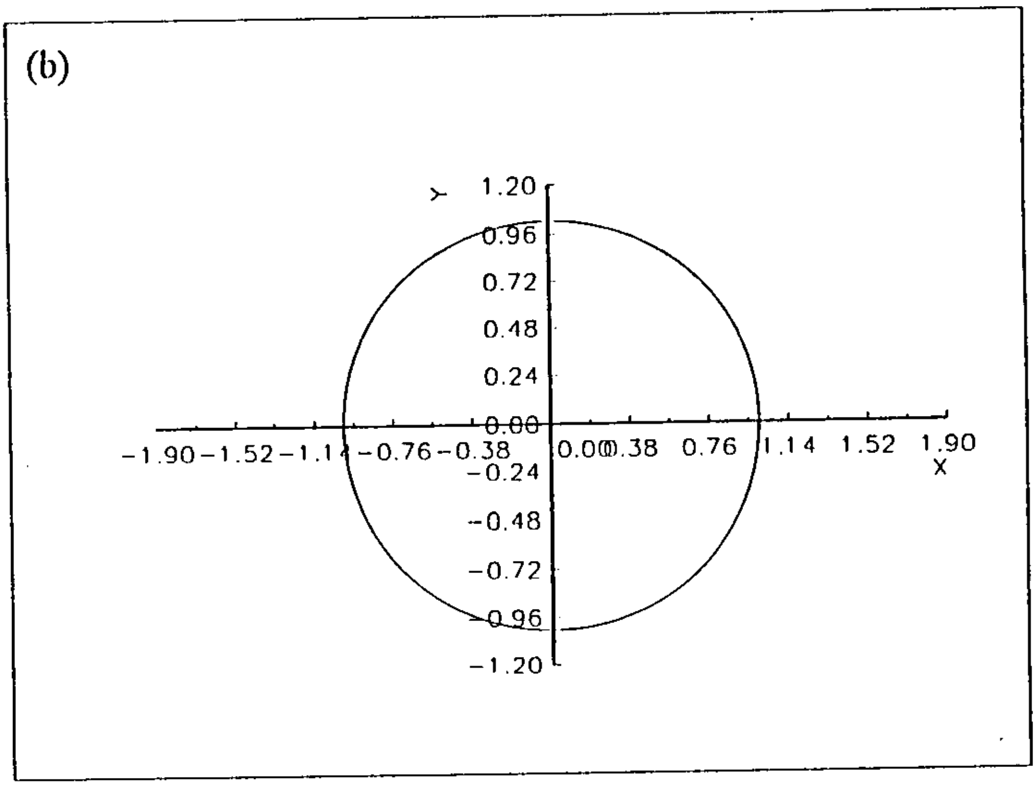
Figure (3.10) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=1, and F=0.5

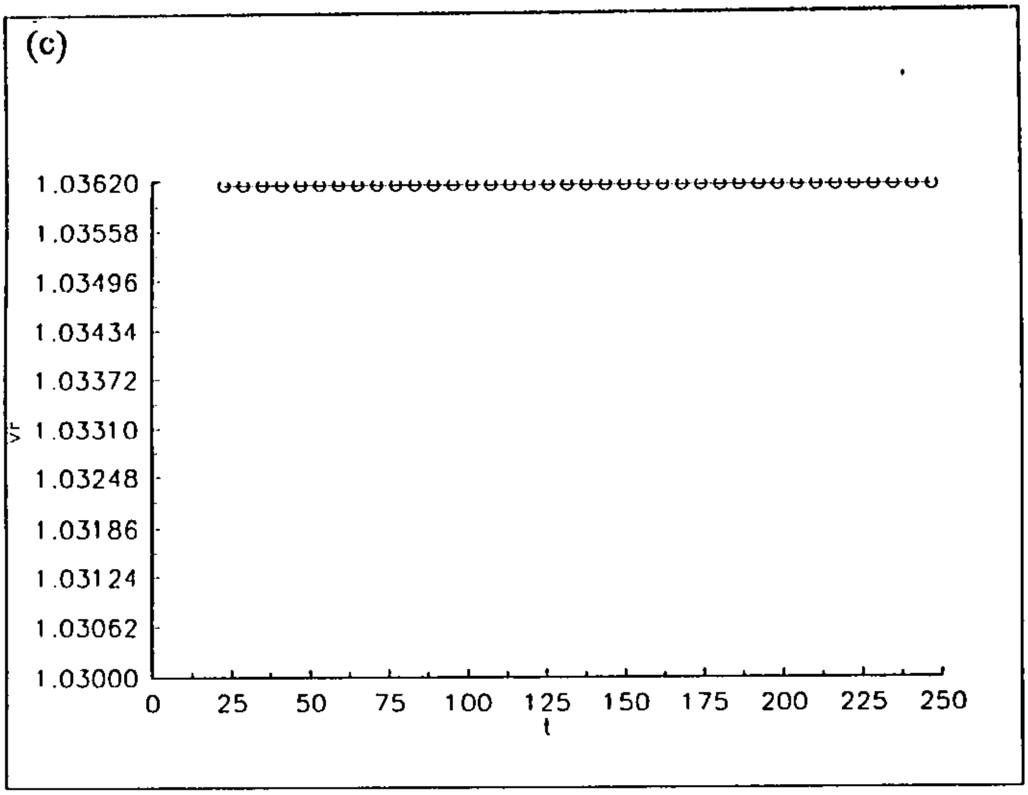


MEAN=0.98979, D=0.2137, PD=21.50%

Figure (3.11) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=1, and F=1


(a)
(0,1,0.1,10,0,1,0)
M=150
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
x
100

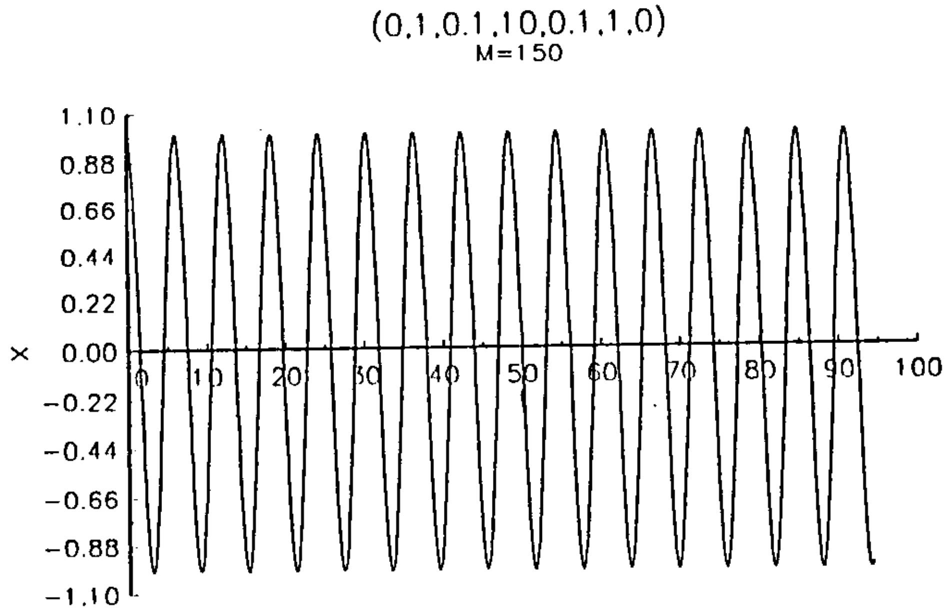

(b)
Y
1.20
0.96
0.72
0.48
0.24
0.00
−0.24
−0.48
−0.72
−0.96
−1.20
−1.90
−1.52
−0.76
−0.38
0.38
0.76
1.14
1.52
1.90
X



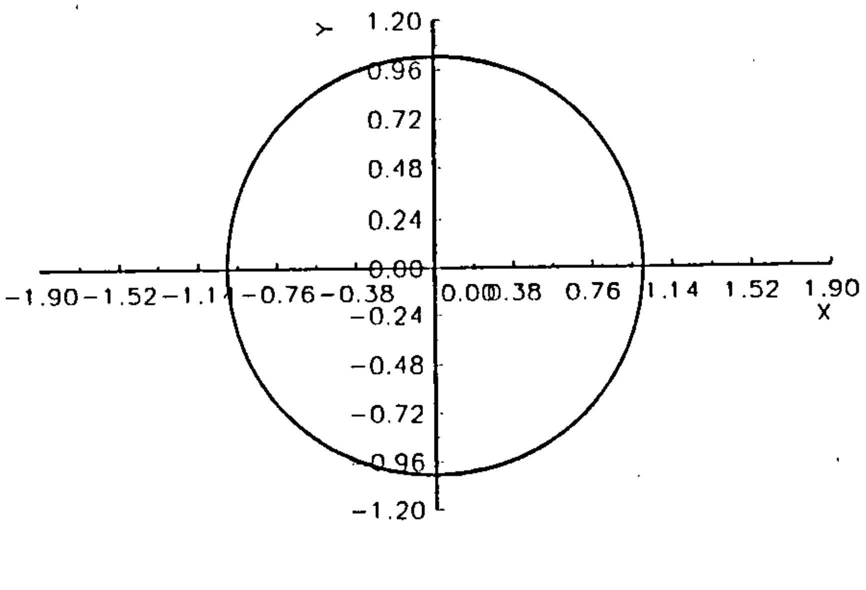
MEAN=1.0362, D=0, PD=0

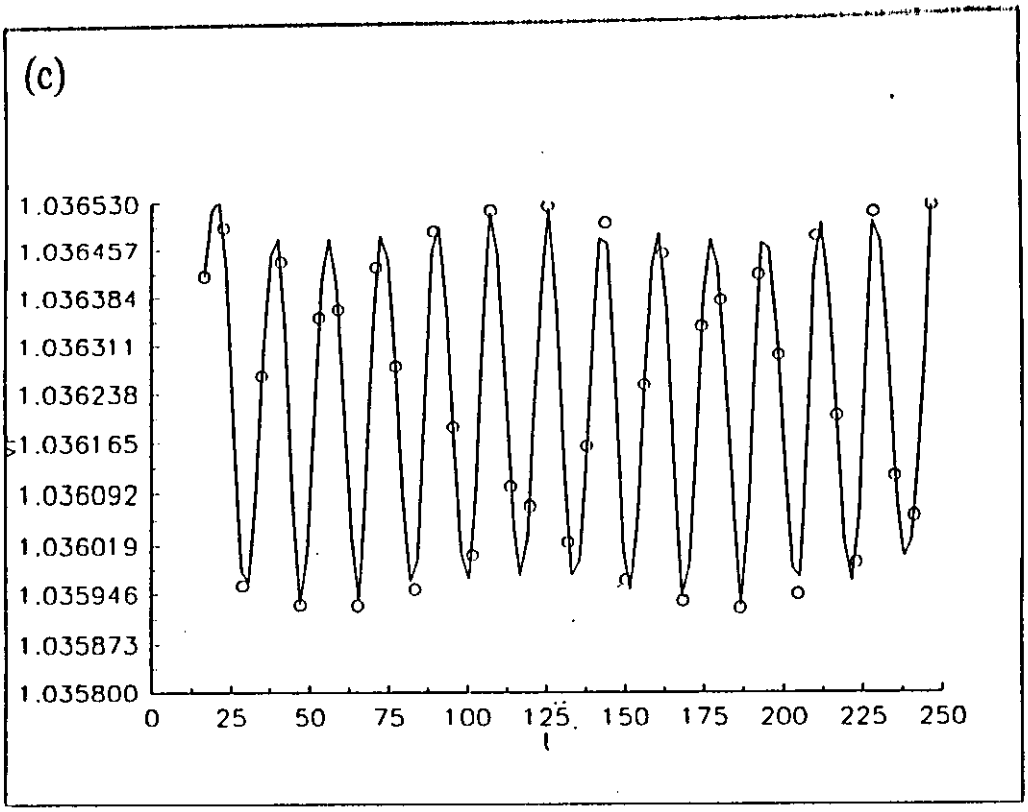
Figure (3.12) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=10, and F=0

(a)


(0,1,0.1,10,0.1,1,0)
M=150
1.10
0.88
0.66
0.44
0.22
X 0.00
-0.22
-0.44
-0.66
-0.88
-1.10
0
10
20
30
40
50
60
70
80
90
100

(b)

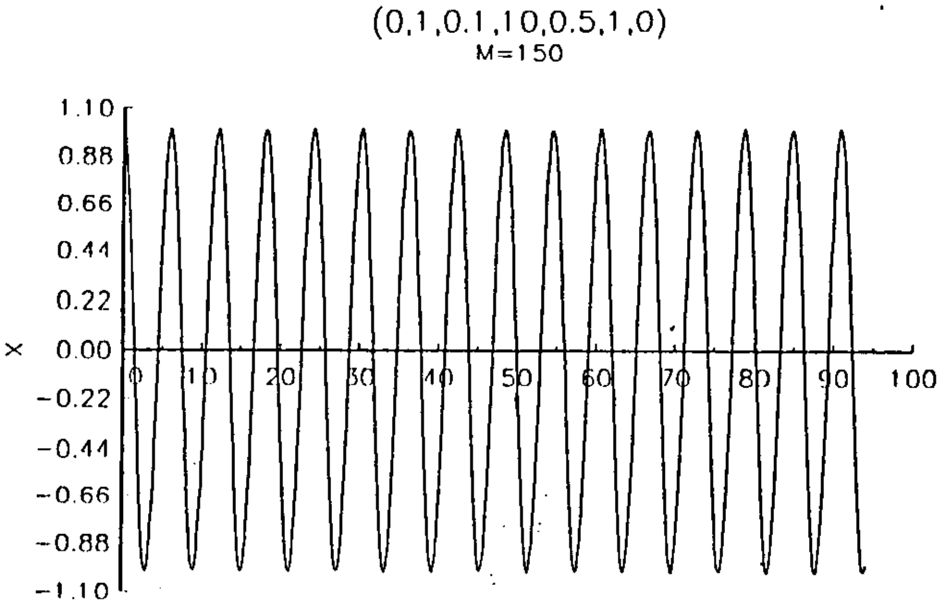

Y
1.20
0.96
0.72
0.48
0.24
0.00
-0.24
-0.48
-0.72
-0.96
-1.20
-1.90
-1.52
-0.76
-0.38
0.38
0.76
1.14
1.52
1.90
X



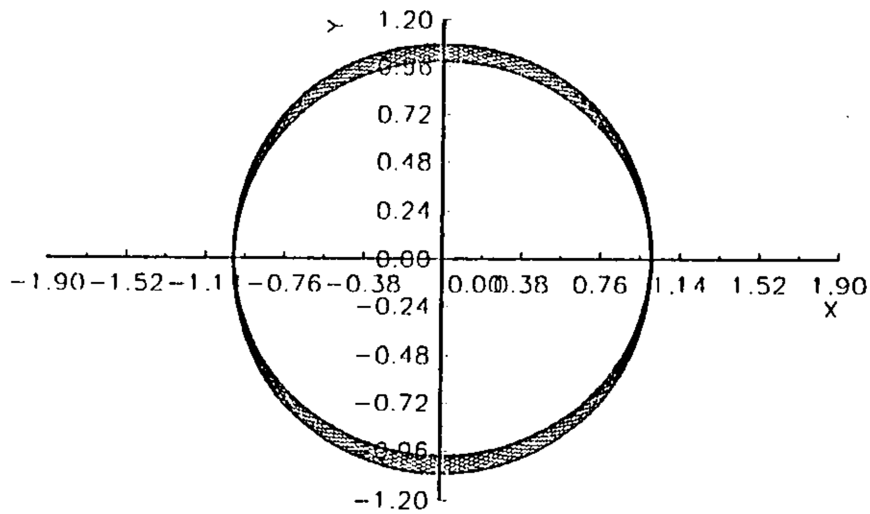
MEAN=1.0362745, D=2.555 E-3, PD=0.02%

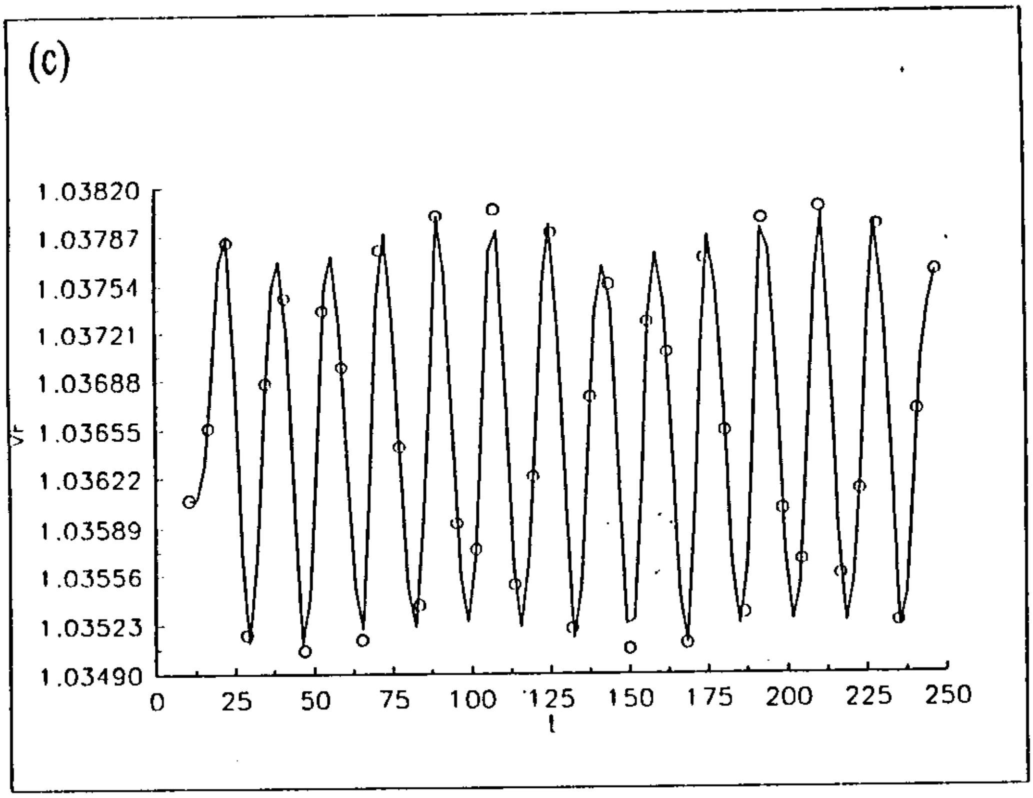
Figure (3.13) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=10, and F=0.1

(a)


(0,1,0.1,10,0.5,1,0)
M=150
1.10
0.88
0.66
0.44
0.22
X
0.00
-0.22
-0.44
-0.66
-0.88
-1.10
100

(b)

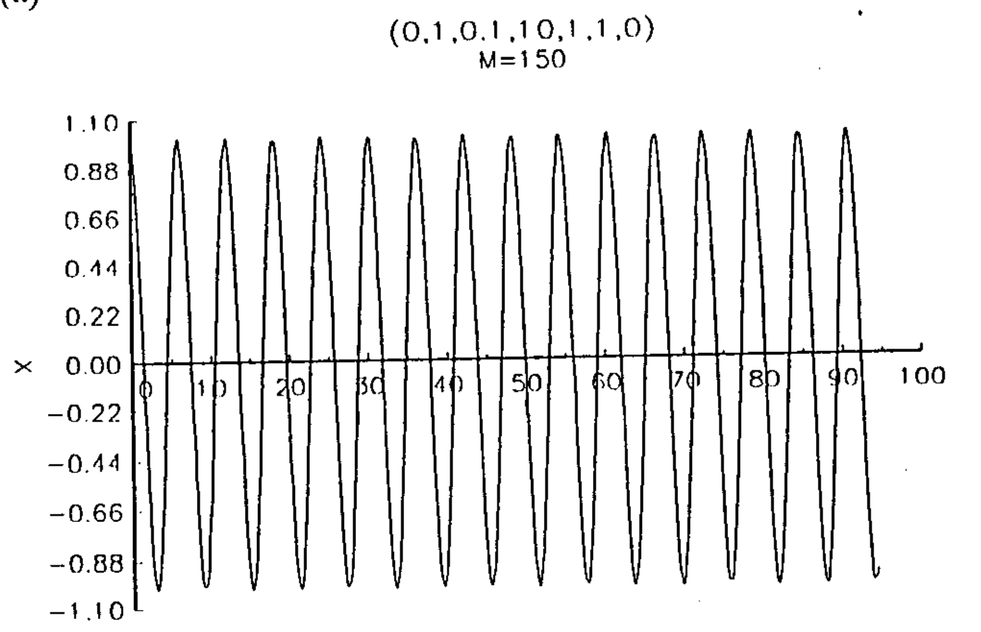

Y
1.20
0.72
0.48
0.24
0.00
-1.90 -1.52
-0.76 -0.38
0.38
0.76
1.14
1.52
1.90
X
-0.24
-0.48
-0.72
-1.20



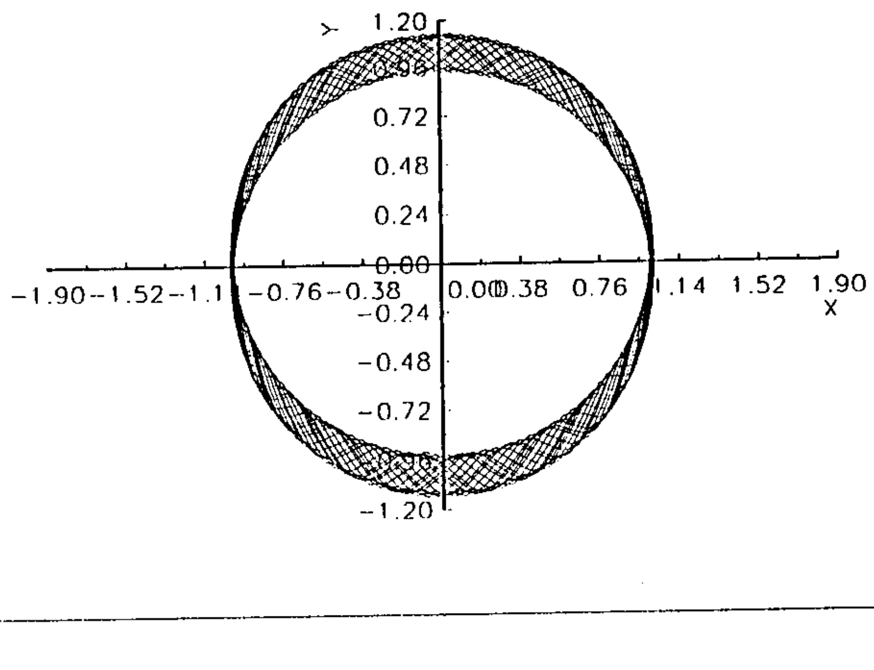
MEAN=1.0366, D=1.4025 E-3, PD=0.14%

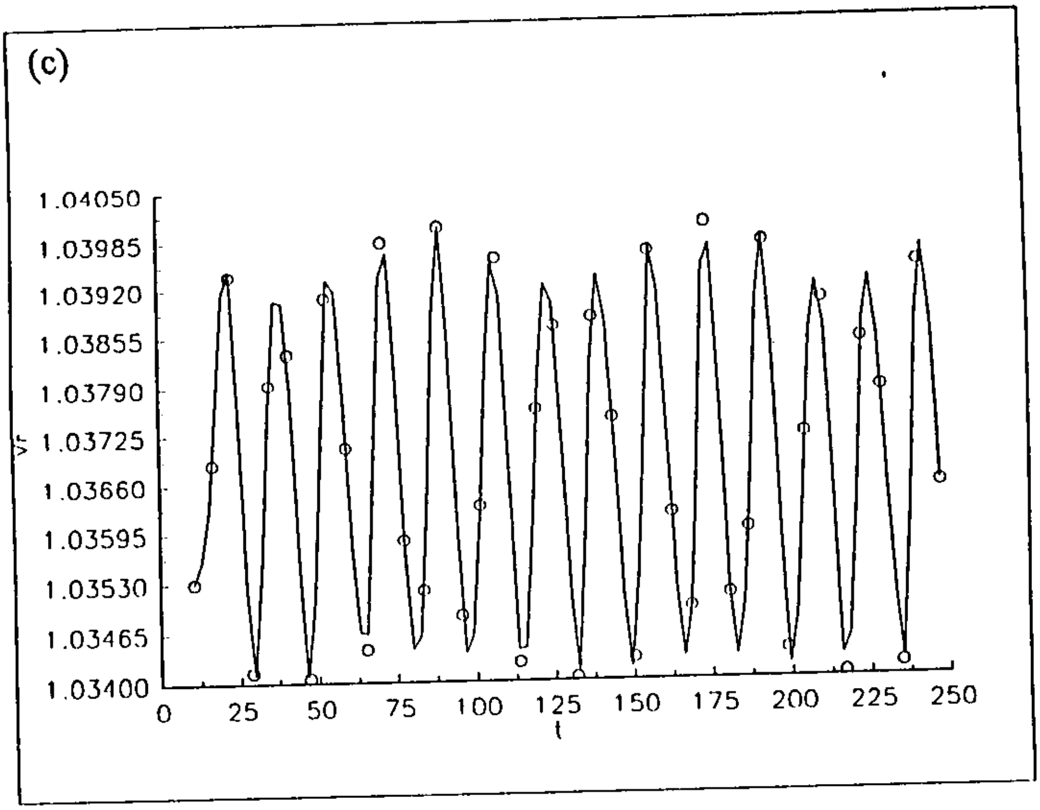
Figure (3.14) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=10, and F=0.5

(a)


(0,1,0.1,10,1,1,0)
M=150
1.10
0.88
0.66
0.44
0.22
x
0.00
−0.22
−0.44
−0.66
−0.88
−1.10
0
10
20
30
40
50
60
70
80
90
100

(b)

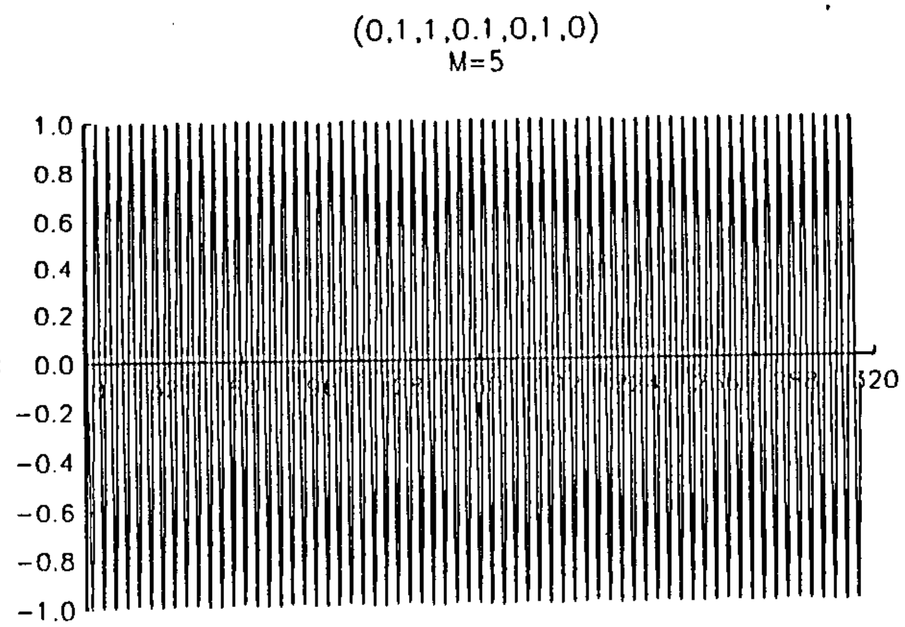

y
1.20
0.72
0.48
0.24
0.00
−0.24
−0.48
−0.72
−1.20
−1.90
−1.52
−0.76
−0.38
0.00
0.38
0.76
1.14
1.52
1.90
x



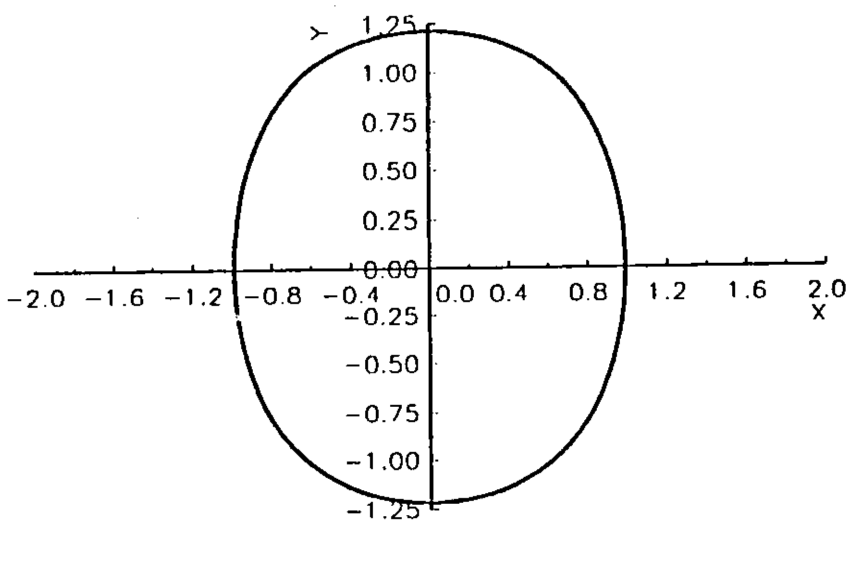
MEAN=1.0373875, D=2.6725 E-3, PD=0.27%

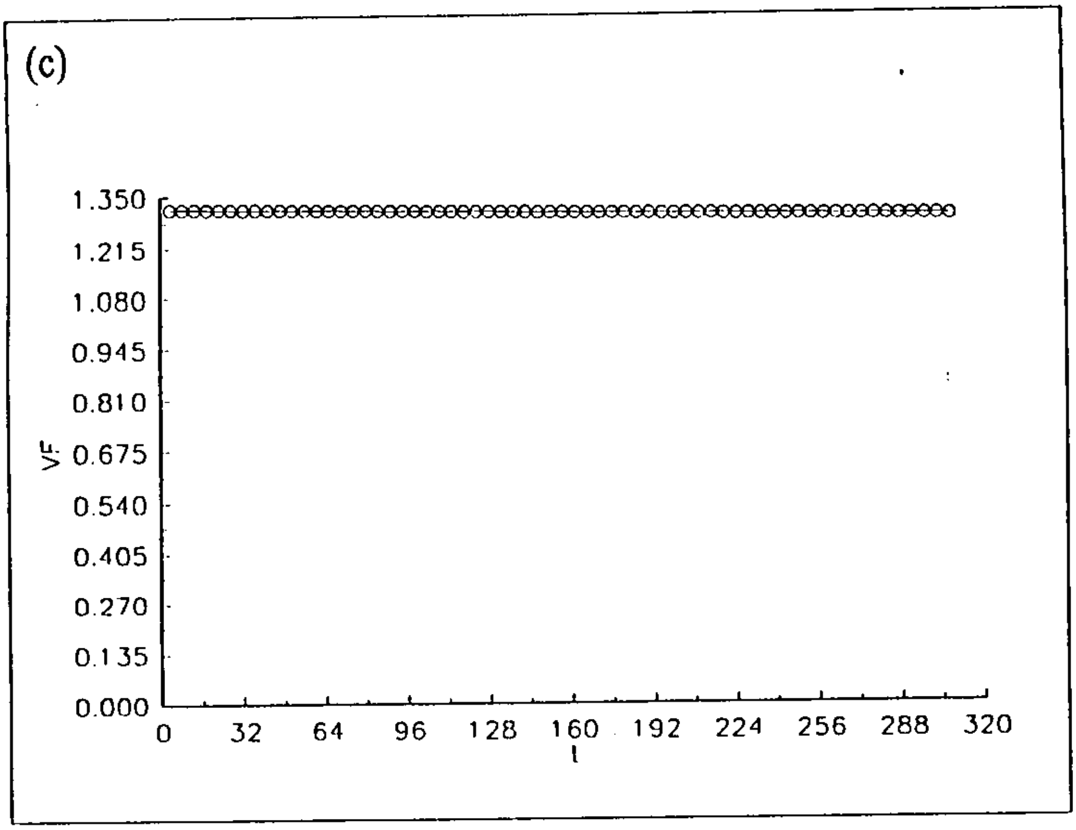
Figure (3.15) Time history, phase plane trajectory, and vibration frequency-time plot for β=0.1, ω=10, and F=1

(a)


(0,1,1,0.1,0,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
X
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
320

(b)

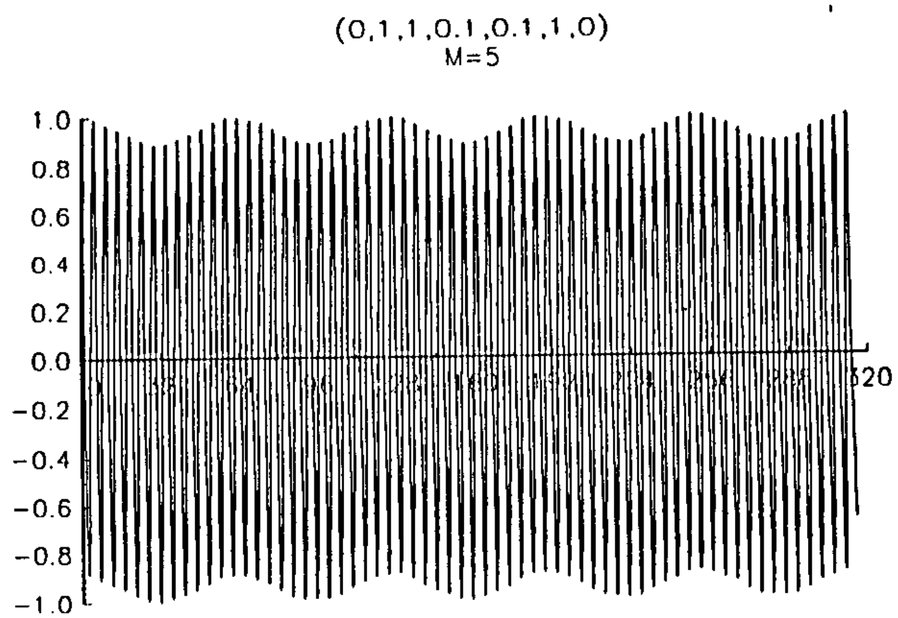

Y
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
-1.25
-2.0 -1.6 -1.2 -0.8 -0.4 0.0 0.4 0.8 1.2 1.6 2.0
X



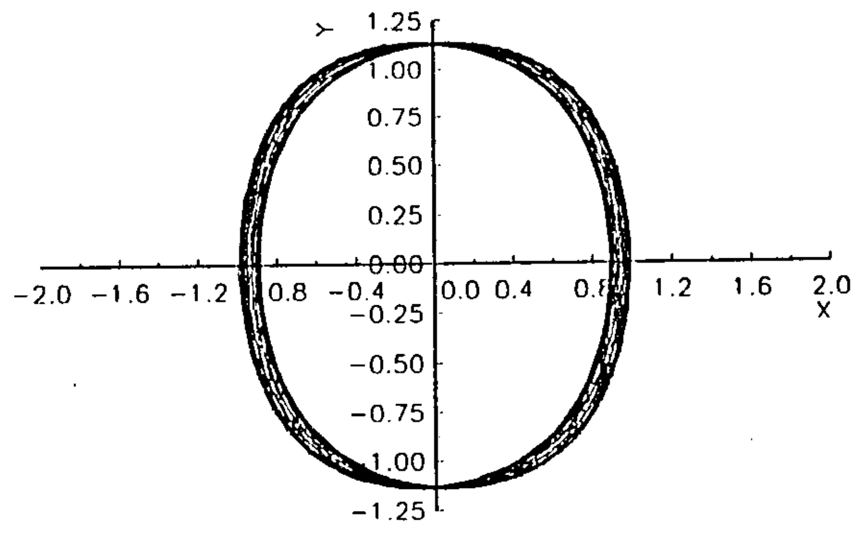
MEAN=1.316, D=0, PD=0

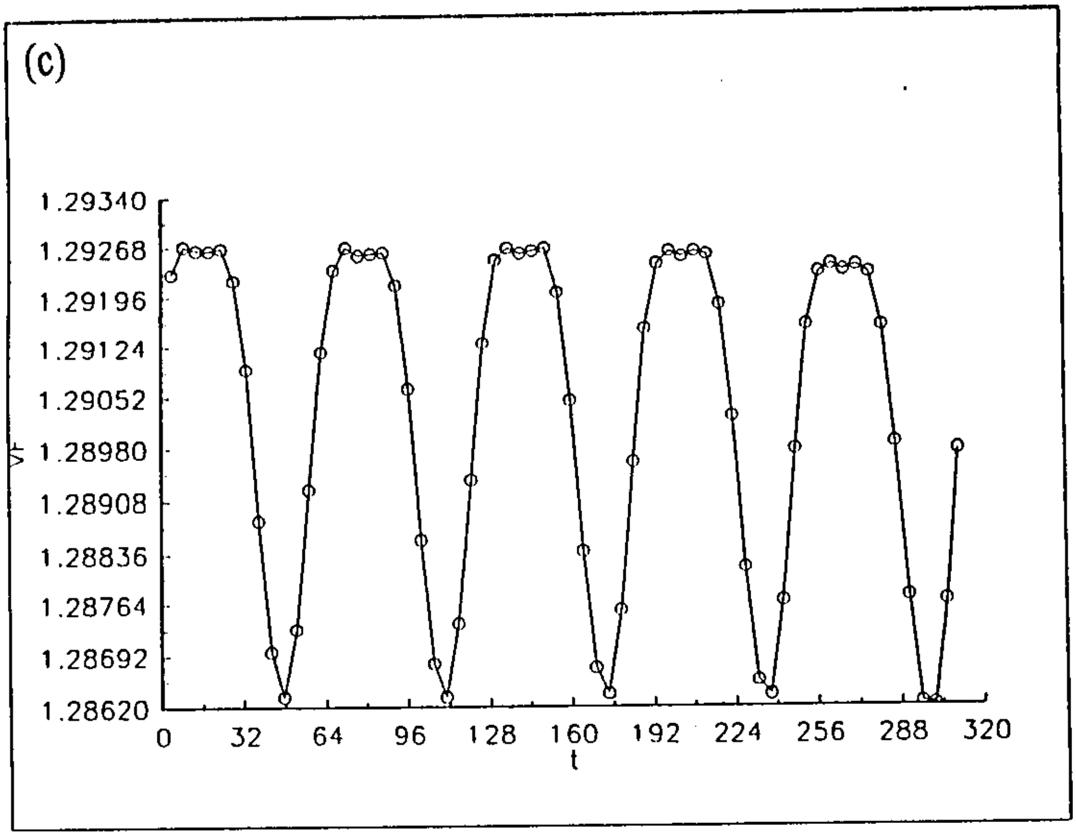
Figure (3.16) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=0.1, and F=0

(a)


(0,1,1,0.1,0.1,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
x
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
320

(b)

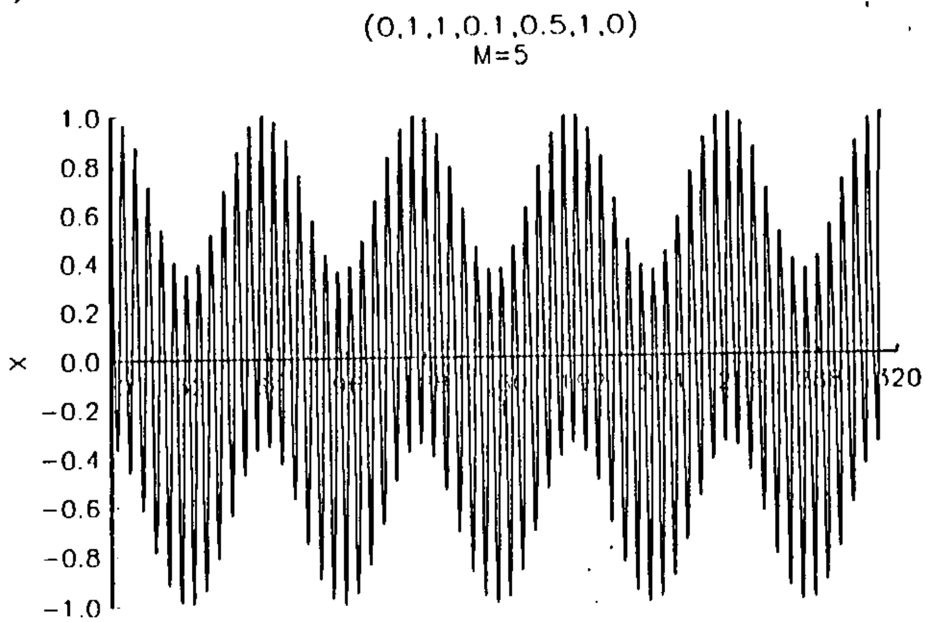

Y
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
-1.25
-2.0
-1.6
-1.2
0.8
-0.4
0.0
0.4
1.2
1.6
2.0
X



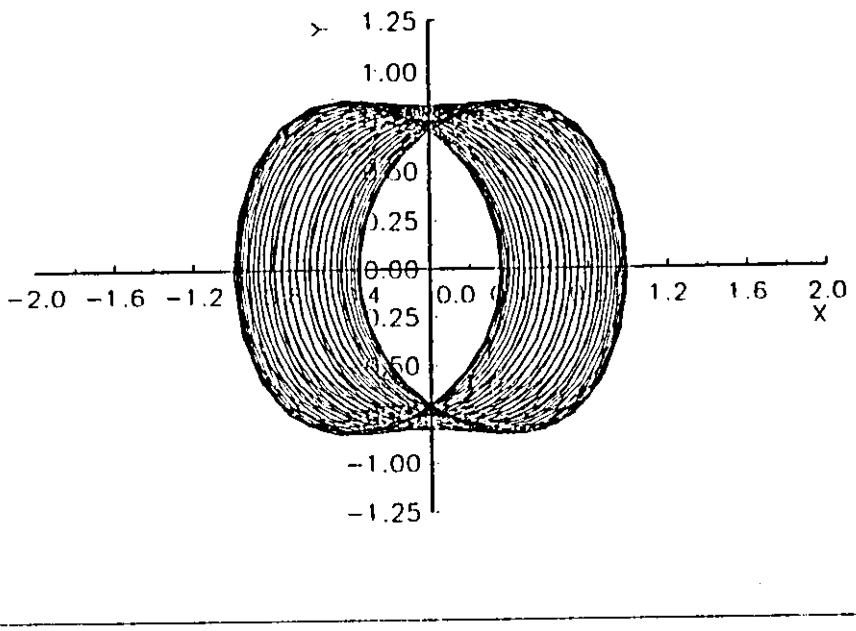
MEAN=1.2894; D=3.24E-3, PD=0.25%

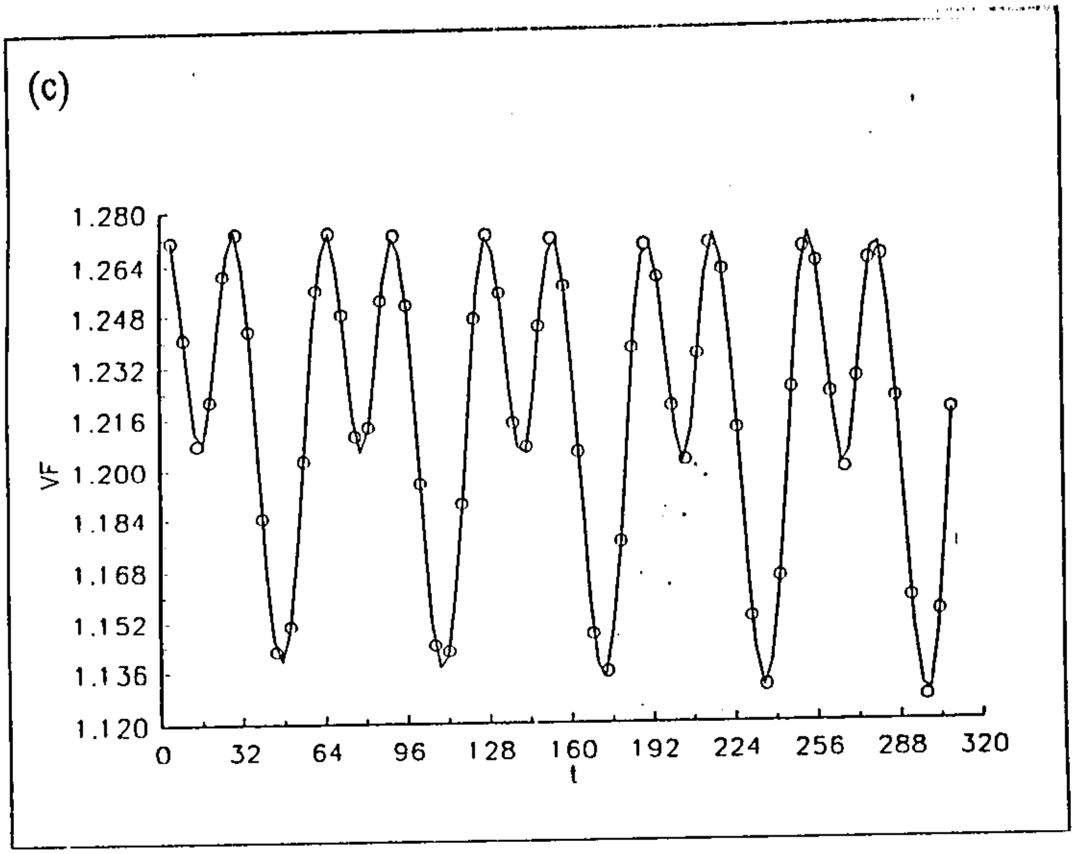
Figure (3.17) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=0.1, and F=0.1

(a)


(0,1,1,0.1,0.5,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
x
0.0
−0.2
−0.4
−0.6
−0.8
−1.0

(b)

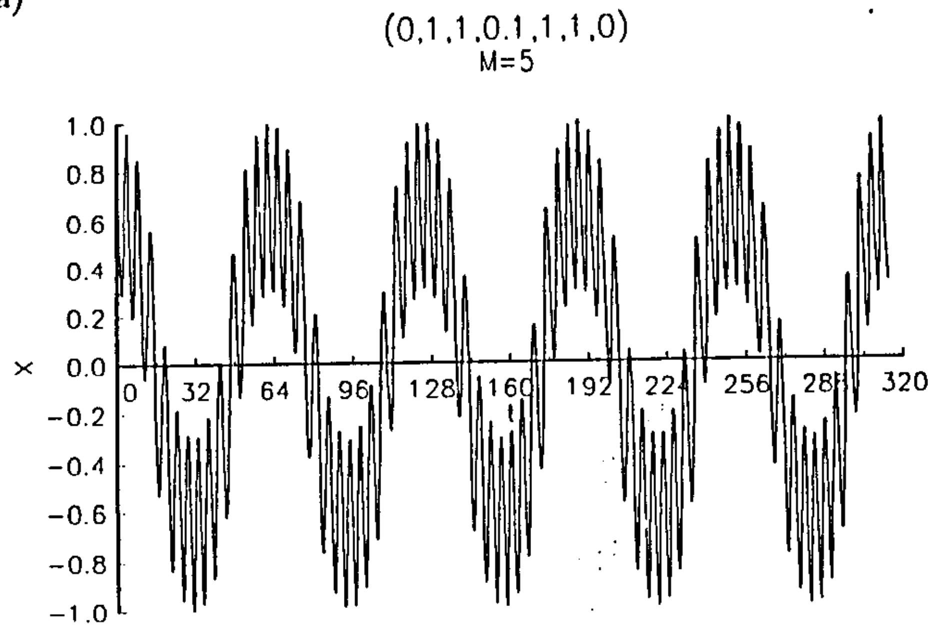

Y
1.25
1.00
0.25
0.00
−1.00
−1.25
−2.0
−1.6
−1.2
0.0
1.2
1.6
2.0
X



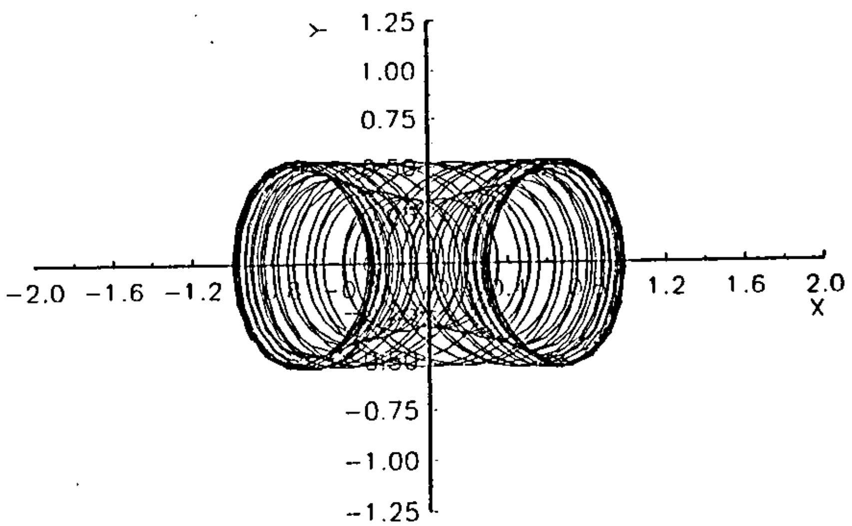
MEAN=1.204, D=0.068, PD=5.65%

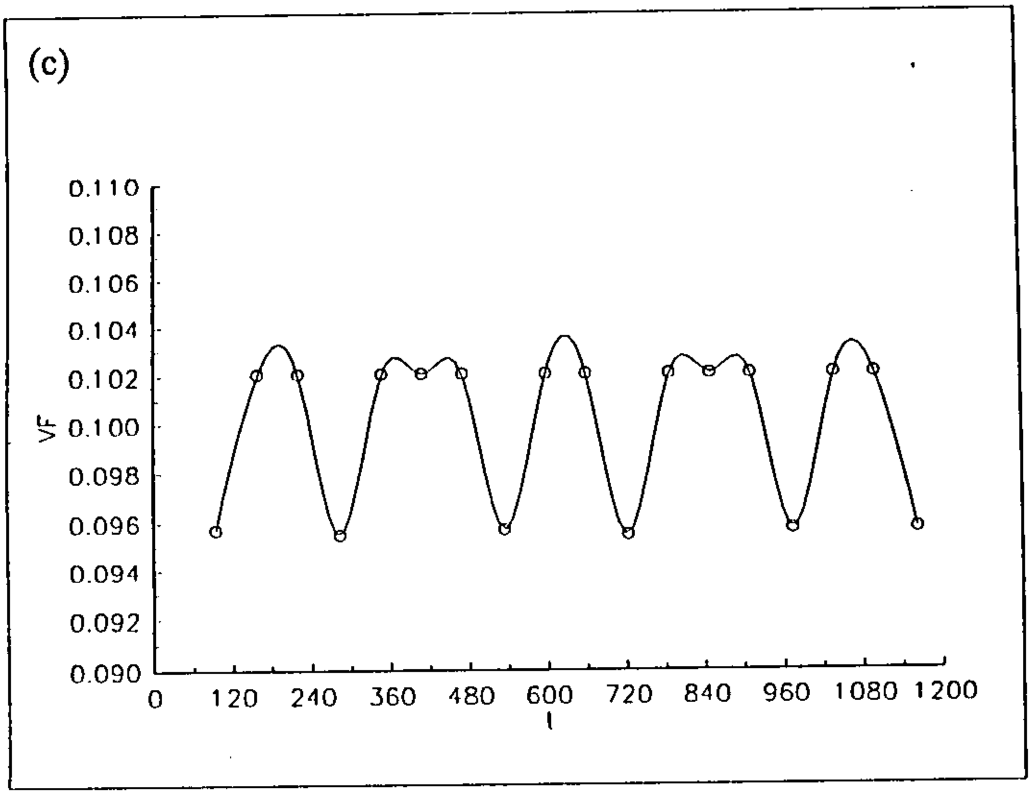
Figure (3.18) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=0.1, and F=0.5

(a)


(0,1,1,0.1,1,1,0)
M=5
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
x
0
32
64
96
128
160
192
224
256
288
320
t

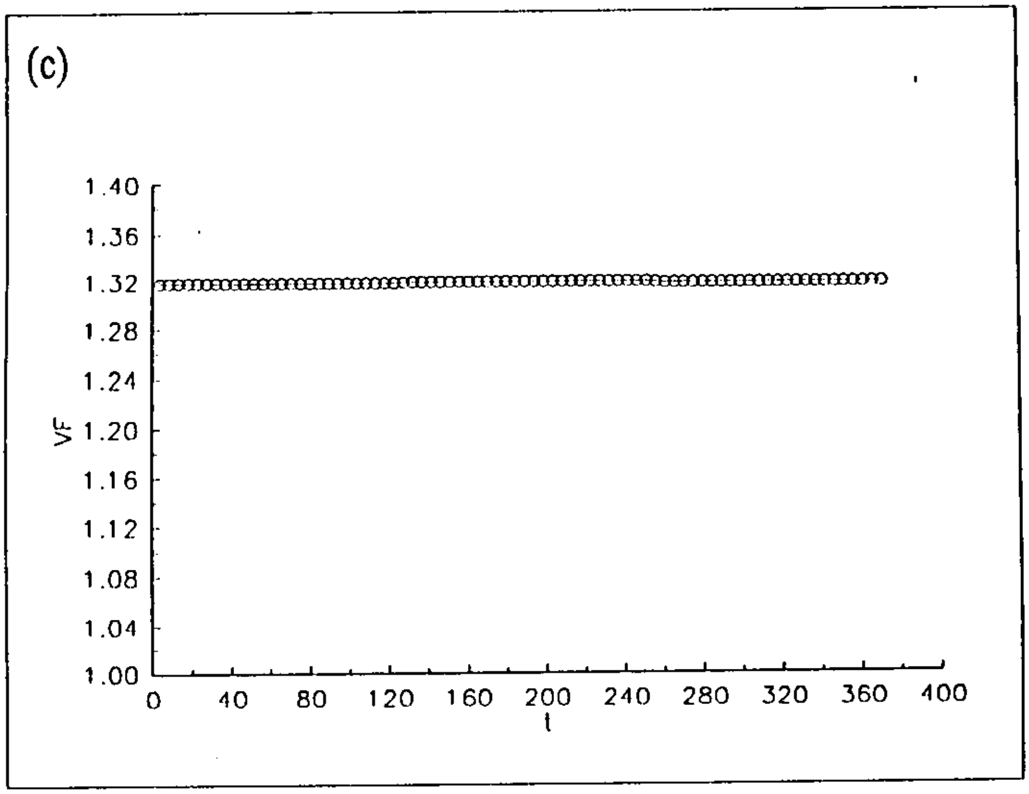
(b)


Y
1.25
1.00
0.75
-0.75
-1.00
-1.25
-2.0
-1.6
-1.2
1.2
1.6
2.0
X



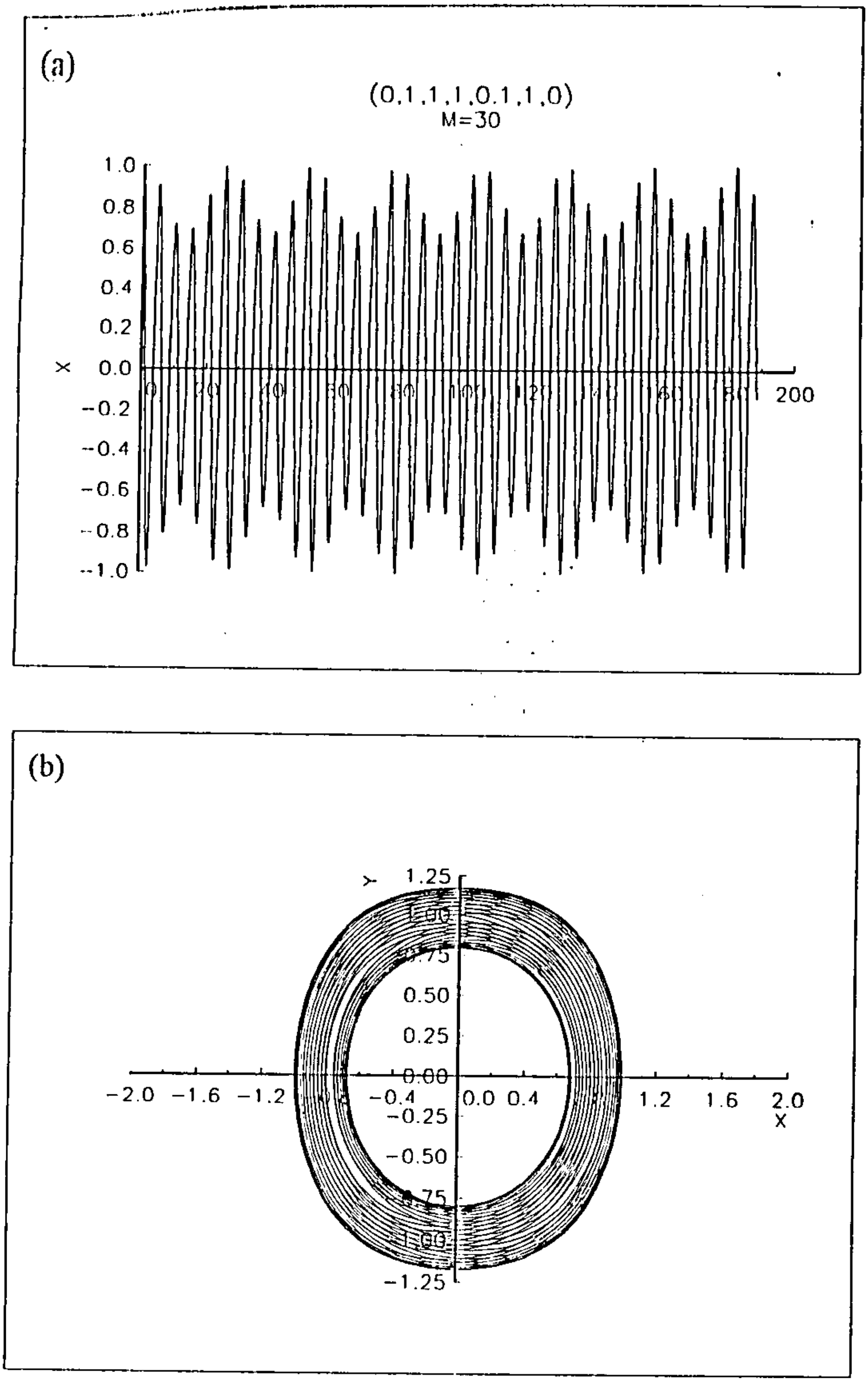
MEAN=0.1104, D=7.1755 E-3, PD=6.49%

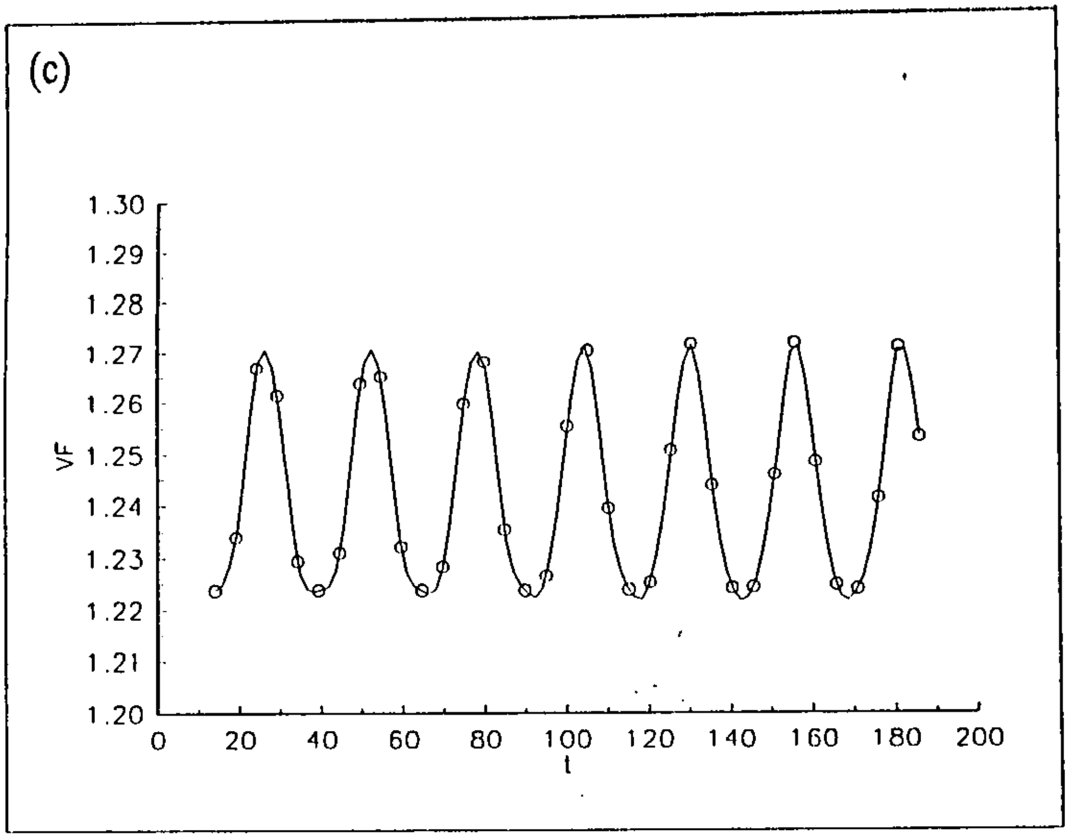
Figure (3.19) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=0.1, and F=1



MEAN=1.32, D=0, PD=0

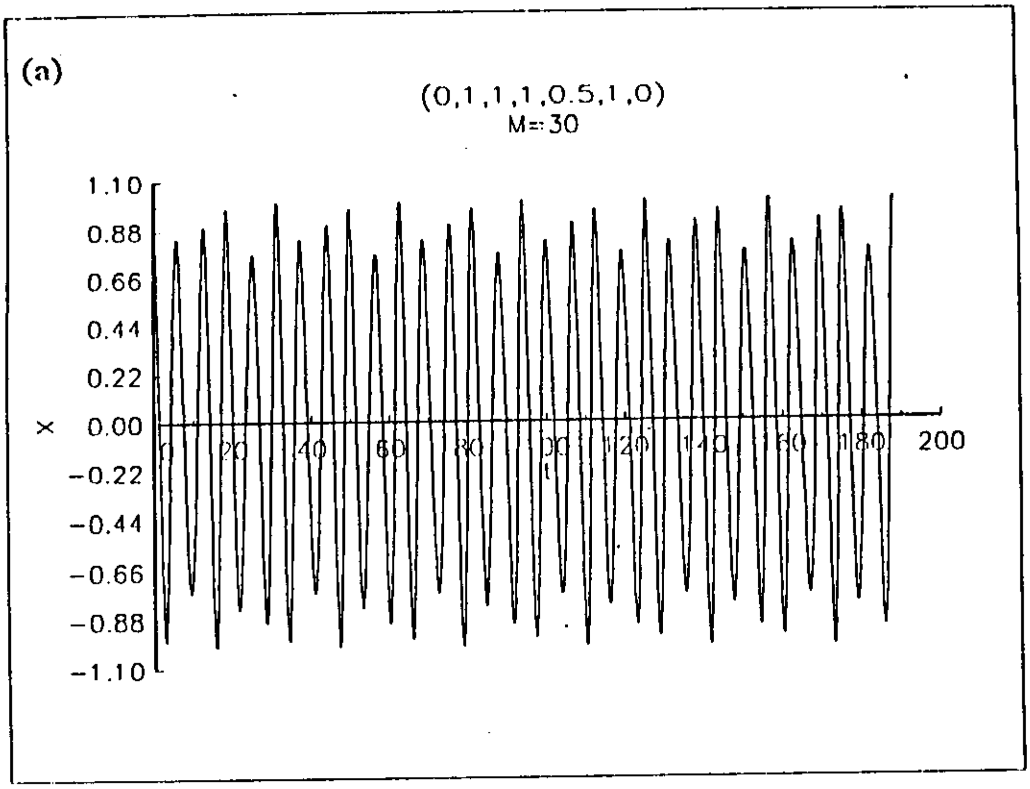
Figure (3.20) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=1, and F=0

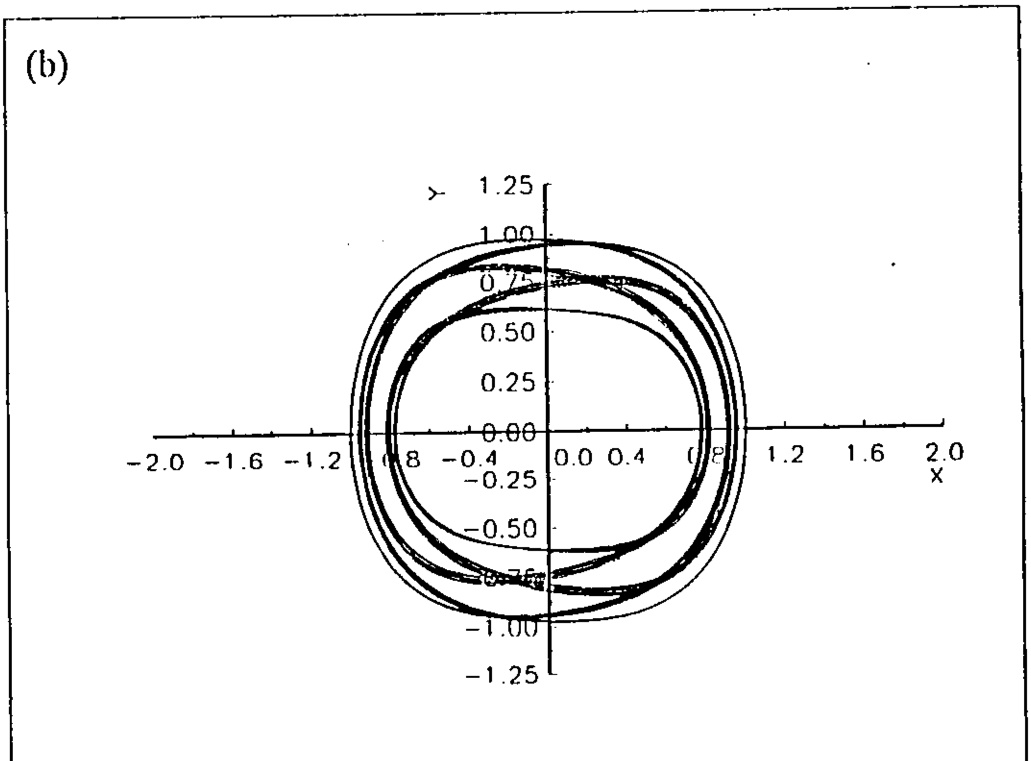

(a)
(0,1,1,1,0.1,1,0)
M=30
x
200
(b)
Y
X

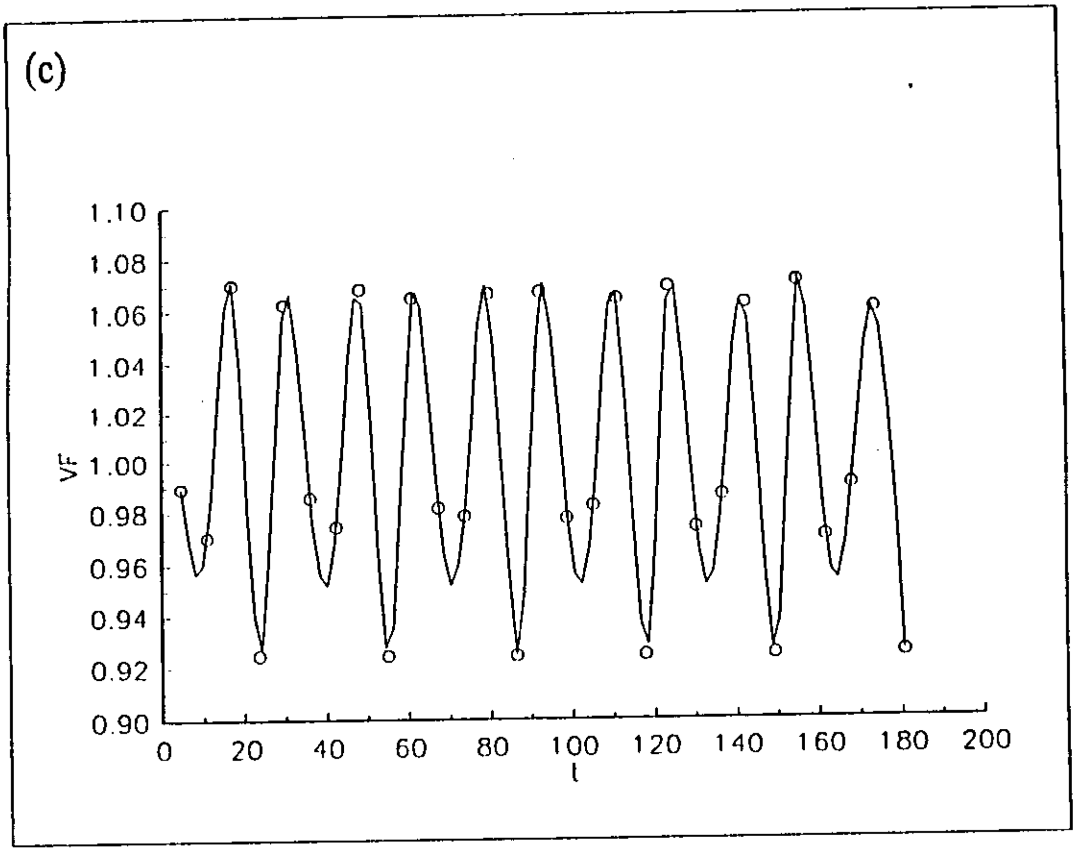


MEAN=1.2475, D=0.0225, PD=1.8%

Figure (3.21) Time history, phase plane trajectory, and vibration frequency-time plot for $\beta=1$, $\omega=1$, and F=0.1

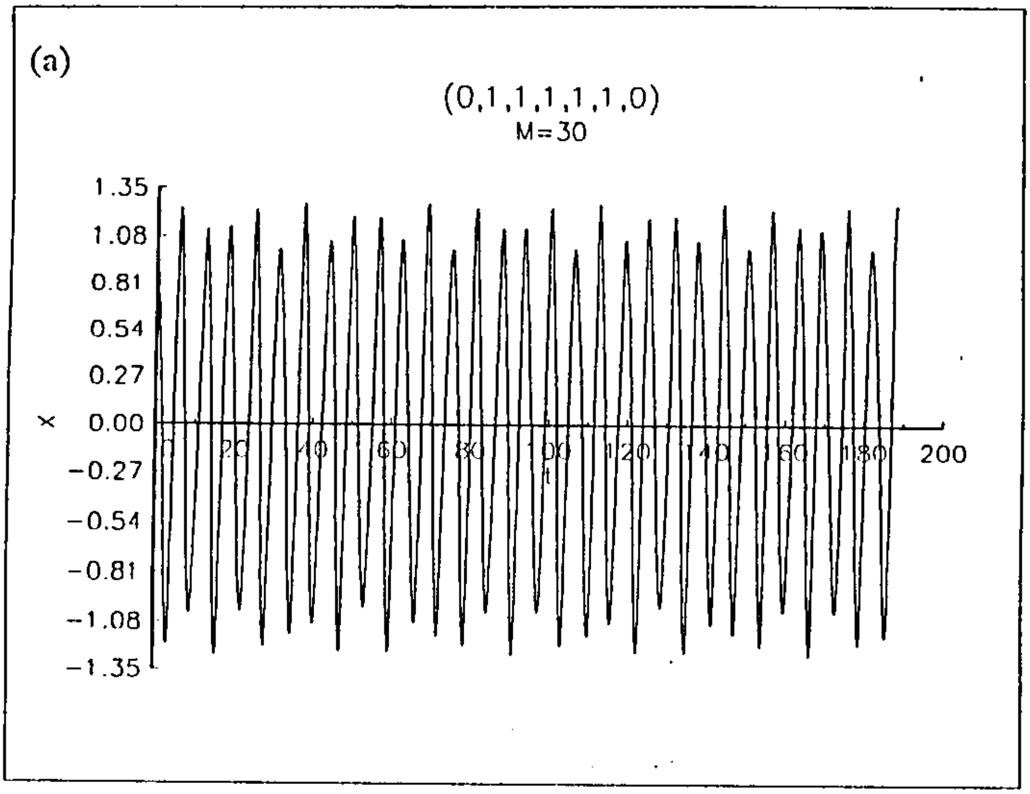

(a)
(0,1,1,1,0.5,1,0)
M=30
1.10
0.88
0.66
0.44
0.22
x
0.00
-0.22
-0.44
-0.66
-0.88
-1.10
200

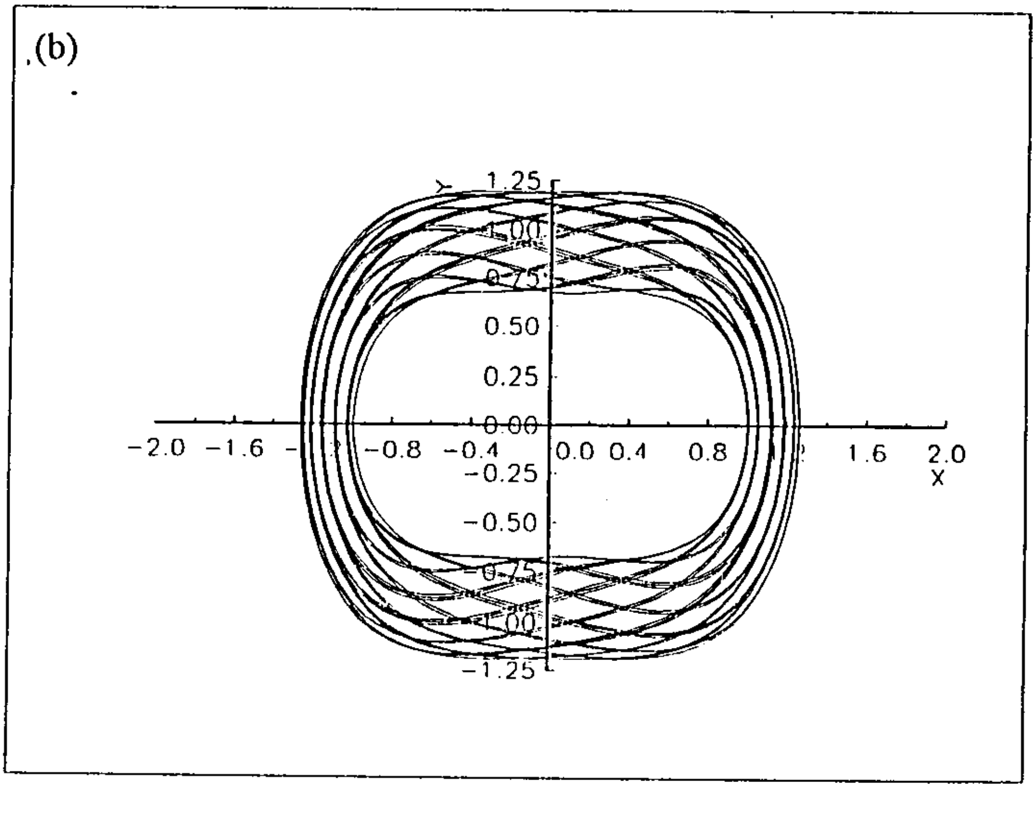

(b)
y
1.25
1.00
0.50
0.25
0.00
-0.25
-0.50
-1.00
-1.25
-2.0
-1.6
-1.2
-0.4
0.0 0.4
1.2
1.6
2.0
x

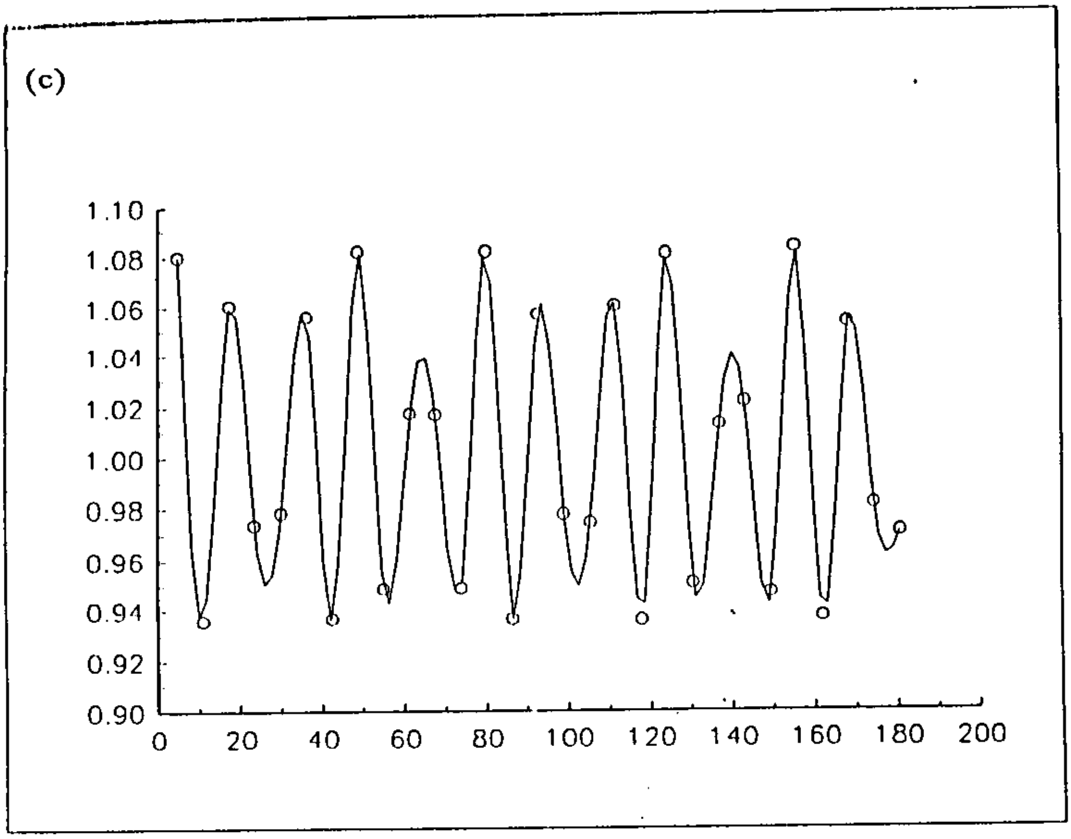


MEAN=0.99, D=0.06, PD=6.06%

Figure (3.22) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=1, and F=0.5

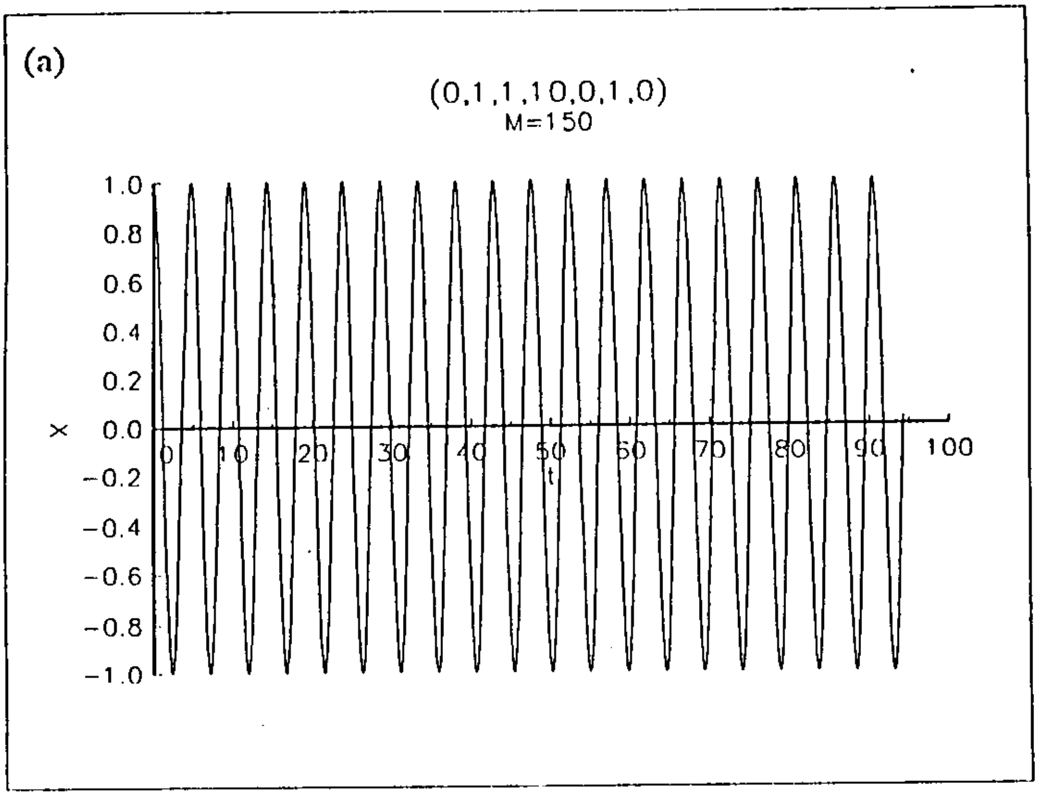

(a)
(0,1,1,1,1,1,0)
M=30
1.35
1.08
0.81
0.54
0.27
x
0.00
-0.27
-0.54
-0.81
-1.08
-1.35
200
t

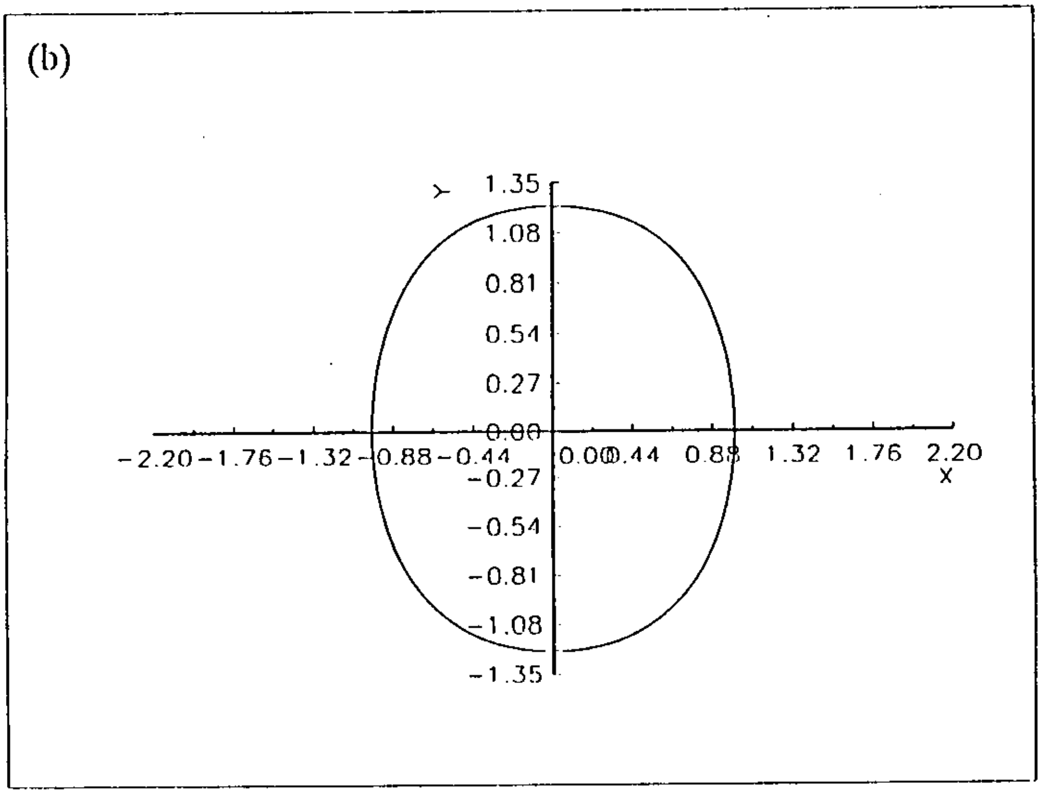

(b)
y
1.25
0.50
0.25
-0.25
-0.50
-1.25
-2.0 -1.6
-0.8 -0.4
0.0 0.4
0.8
1.6
2.0
x

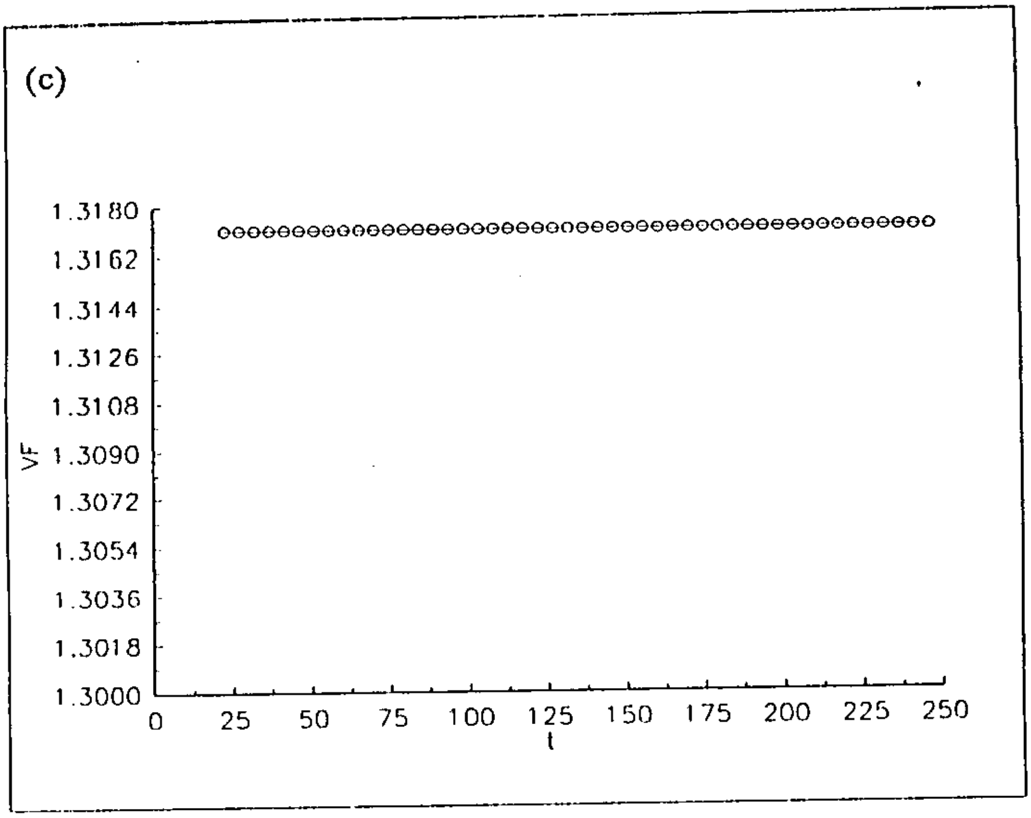


MEAN=1.015, D=0.075, PD=7.36%

Figure (3.23) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=1, and F=1


(a)
(0,1,1,10,0,1,0)
M=150
x
t
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
10
20
30
40
50
60
70
80
90
100

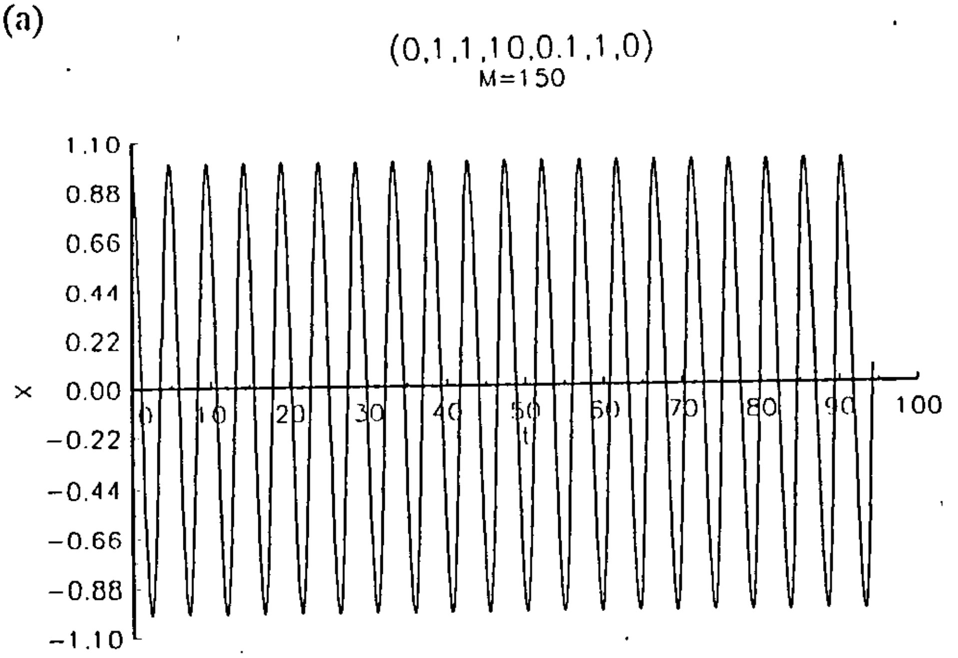

(b)
Y
X
1.35
1.08
0.81
0.54
0.27
0.00
−0.27
−0.54
−0.81
−1.08
−1.35
−2.20
−1.76
−1.32
−0.88
−0.44
0.00
0.44
0.88
1.32
1.76
2.20



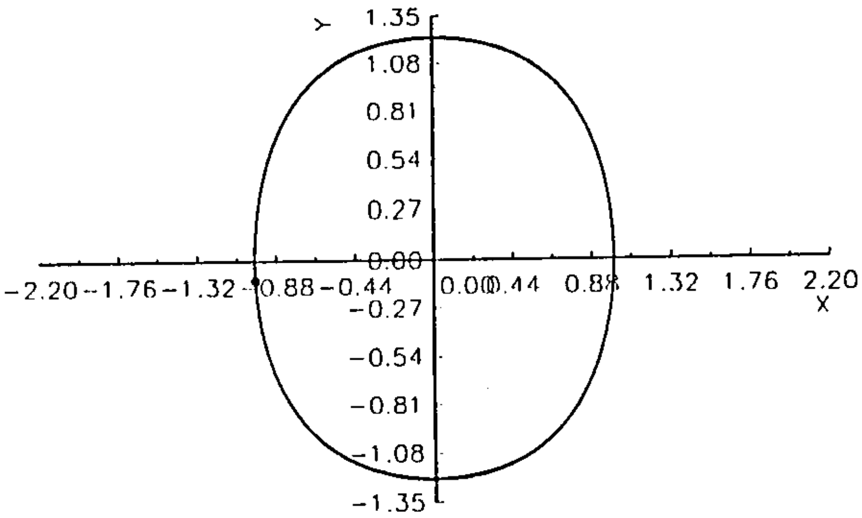
MEAN=1.3172, D=0, PD=0

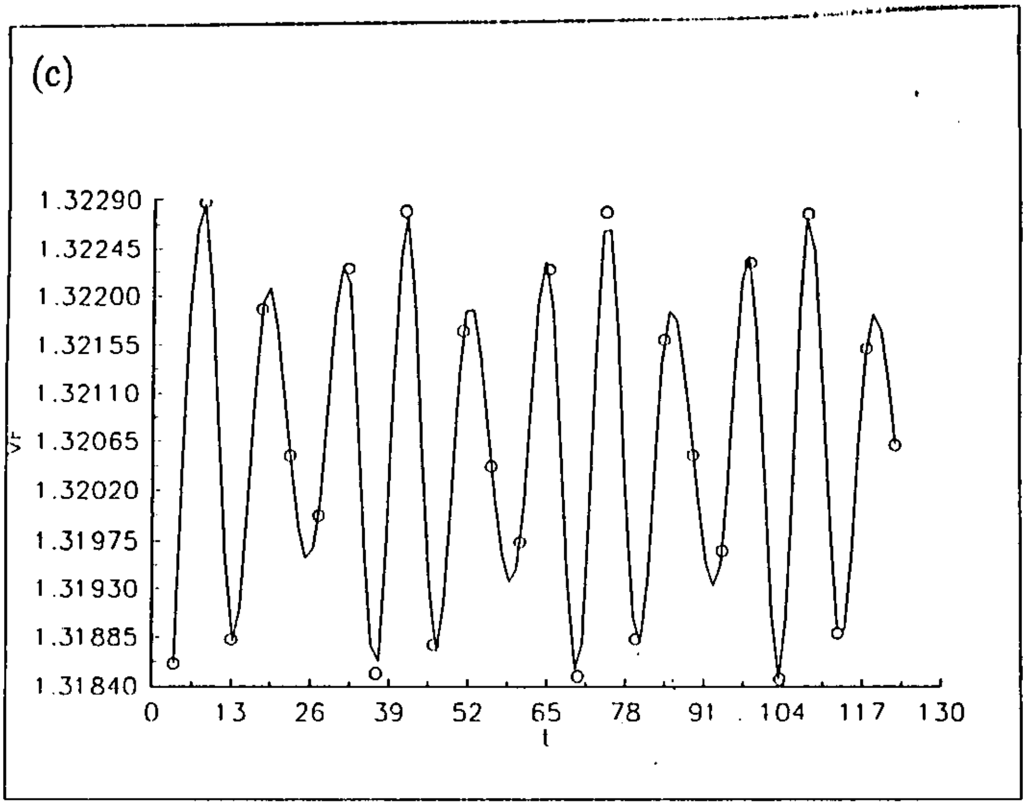
Figure (3.24) Time history, phase plane trajectory, and vibration frequency-time plot for $\beta=1$, $\omega=10$, and $F=0$

(a)


(0,1,1,10,0.1,1,0)
M=150
1.10
0.88
0.66
0.44
0.22
x 0.00
−0.22
−0.44
−0.66
−0.88
−1.10
0
10
20
30
40
50
60
70
80
90
100
t

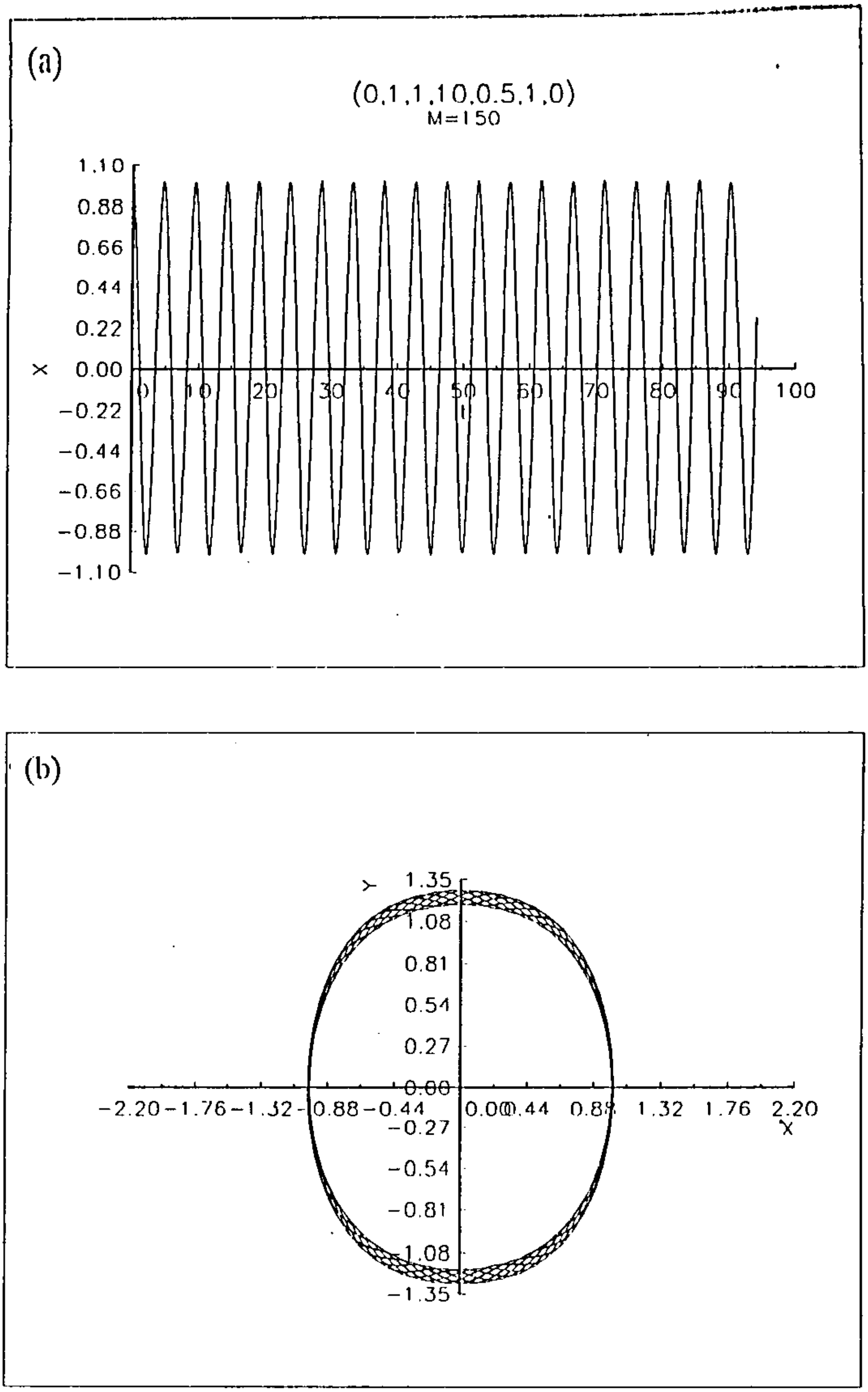
(b)


Y
1.35
1.08
0.81
0.54
0.27
0.00
−0.27
−0.54
−0.81
−1.08
−1.35
−2.20 −1.76 −1.32 −0.88 −0.44
0.00 0.44 0.88 1.32 1.76 2.20
X

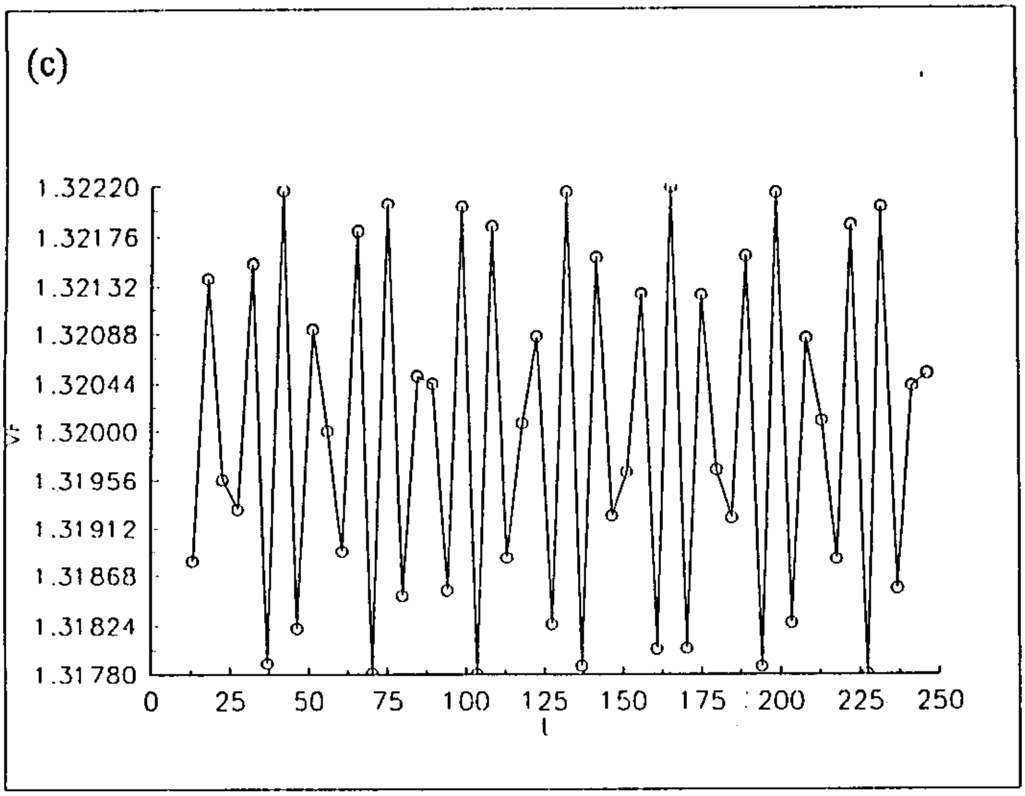


MEAN=1.3007625, D=1.1375 E-3, PD=0.09%

Figure (2.25) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=10, and F=0.1



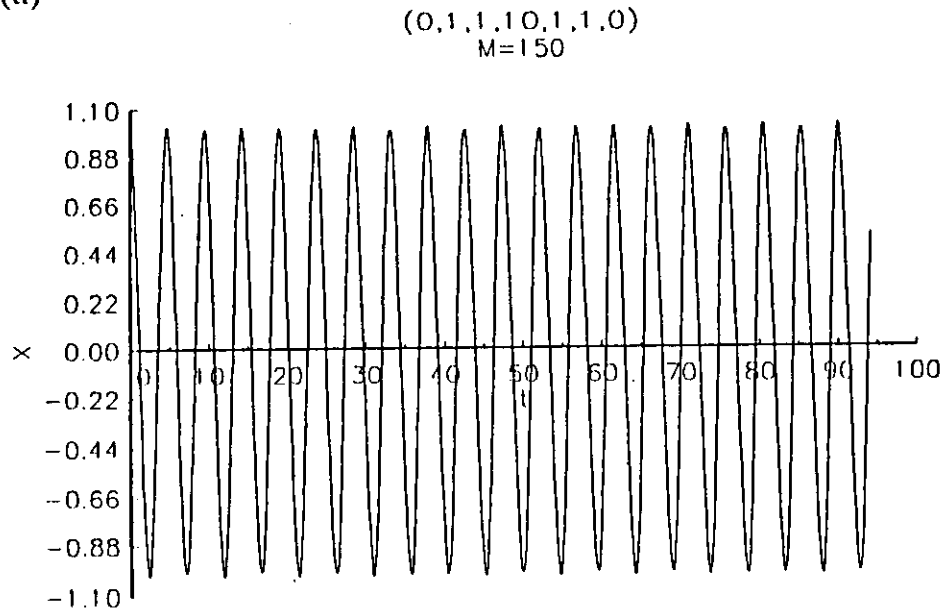
(a)
(0,1,1,10,0.5,1,0)
M=150
1.10
0.88
0.66
0.44
0.22
0.00
-0.22
-0.44
-0.66
-0.88
-1.10
X
0
10
20
30
40
50
60
70
80
90
100
t
(b)
Y
1.35
1.08
0.81
0.54
0.27
0.00
-0.27
-0.54
-0.81
-1.08
-1.35
-2.20
-1.76
-1.32
-0.88
-0.44
0.00
0.44
0.88
1.32
1.76
2.20
X



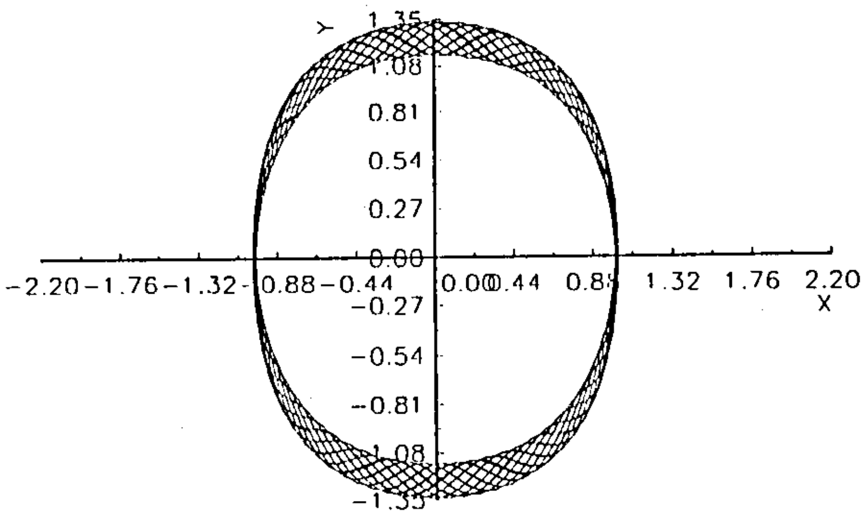
MEAN=1.31989, D=2.09 E-3, PD=0.16%

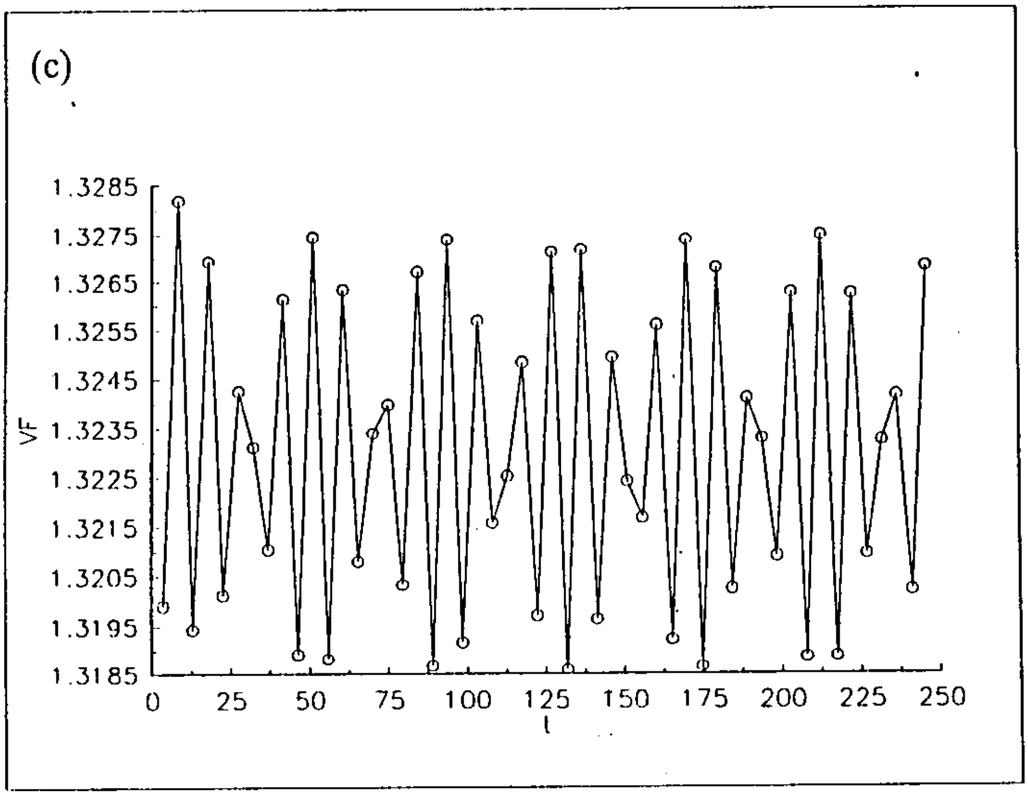
Figure (3.26) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=10, and F=0.5

(a)


(0,1,1,10,1,1,0)
M=150
1.10
0.88
0.66
0.44
0.22
x
0.00
-0.22
-0.44
-0.66
-0.88
-1.10
10
20
30
40
50
60
70
80
90
100
t

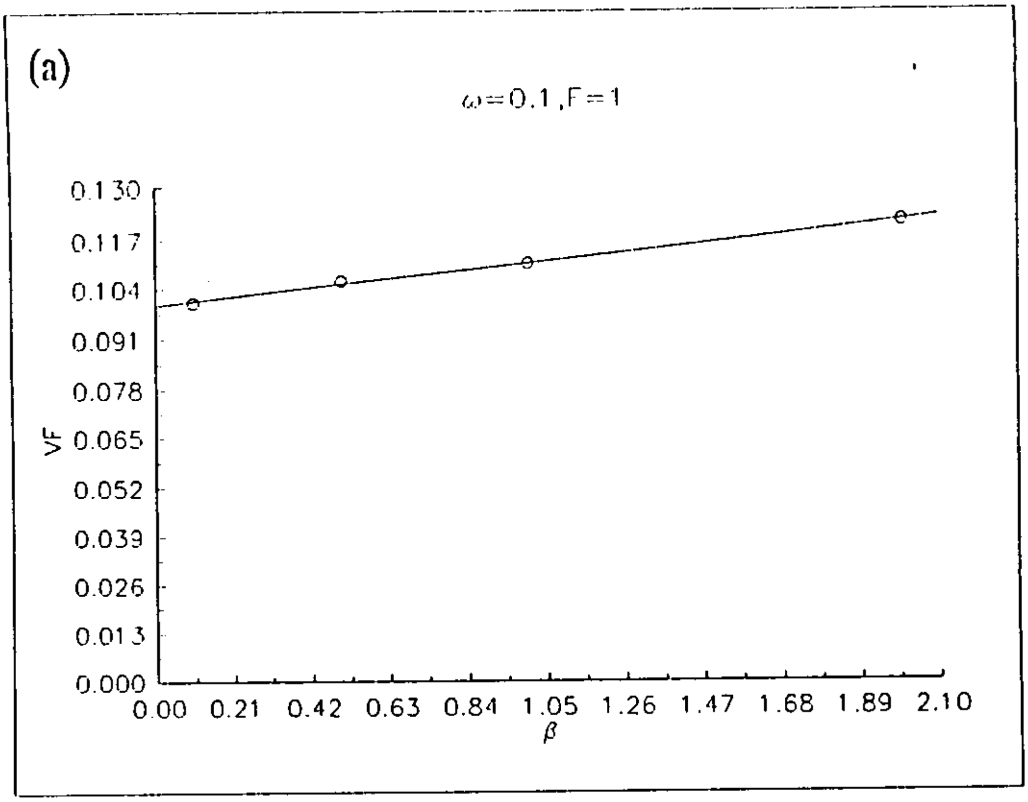
(b)


Y
1.35
1.08
0.81
0.54
0.27
0.00
-0.27
-0.54
-0.81
-2.20
-1.76
-1.32
-0.88
-0.44
0.00
0.44
1.32
1.76
2.20
X



MEAN=1.323, D=4.5 E-3, PD=0.34%

Figure (3.27) Time history, phase plane trajectory, and vibration frequency-time plot for β=1, ω=10, and F=1



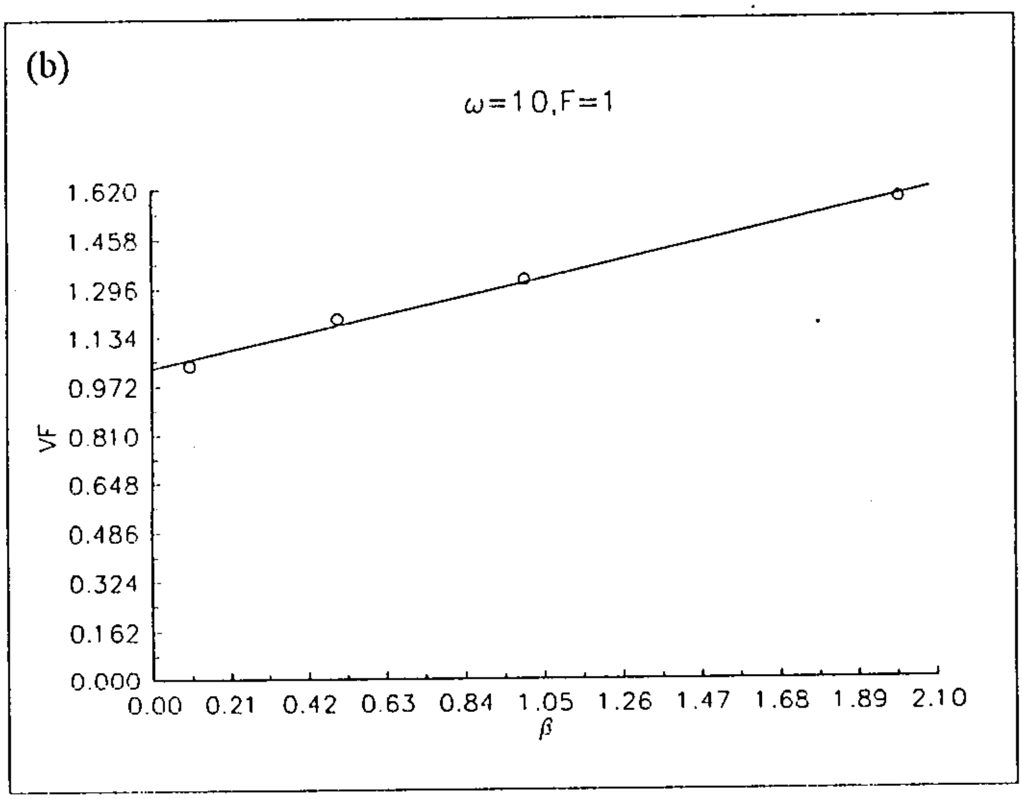
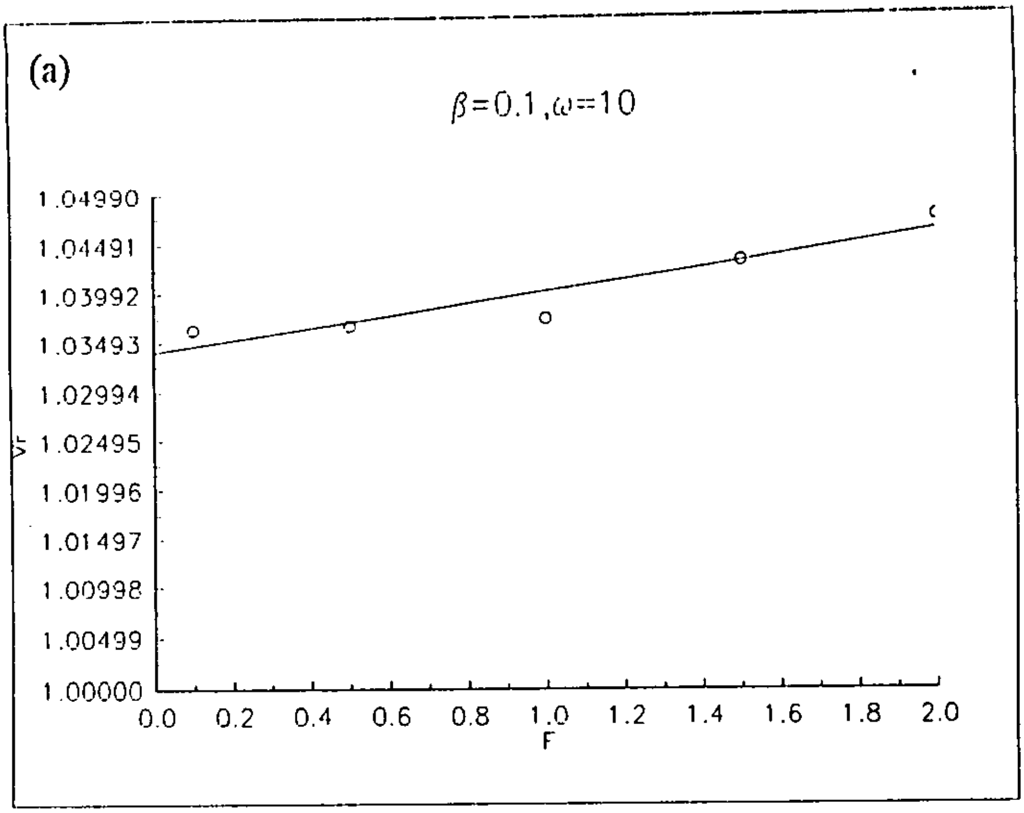


Figure (3.28) Effect of the nonlinear stiffness on the vibration frequency



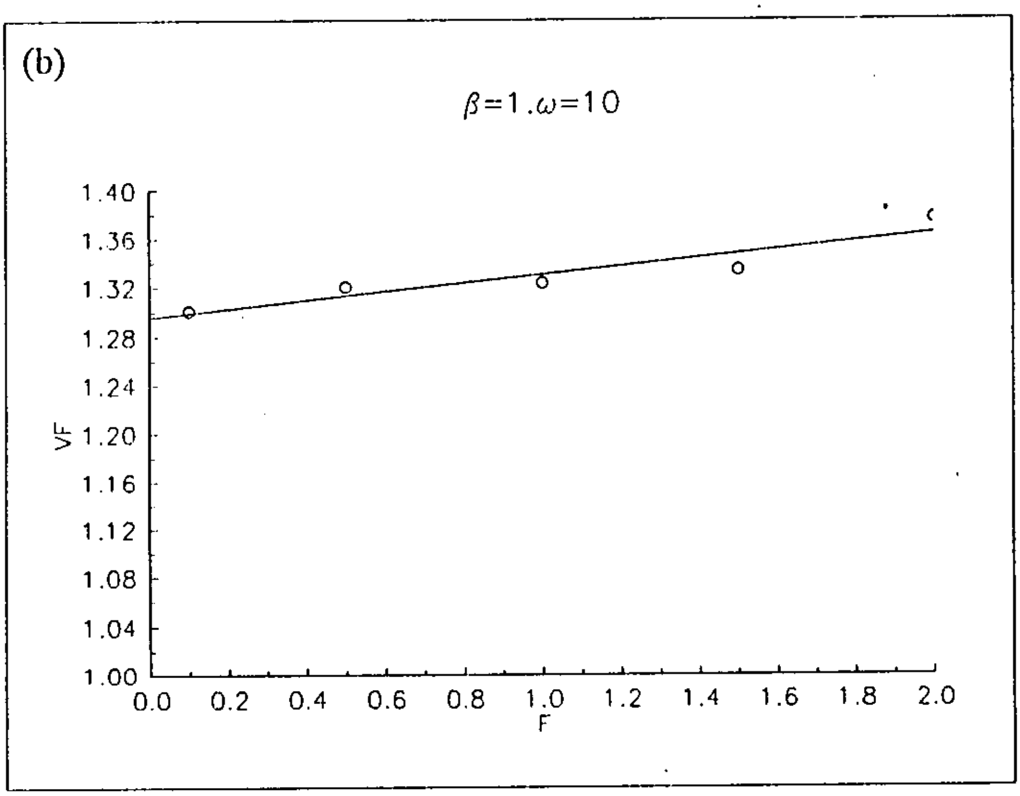
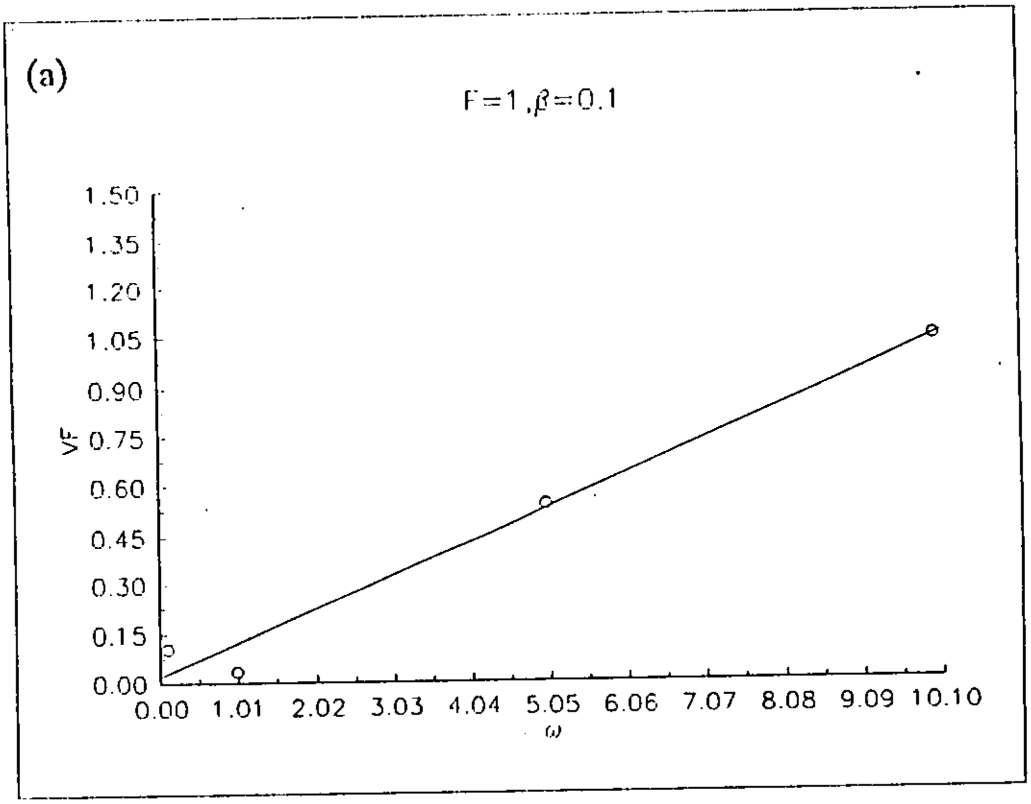
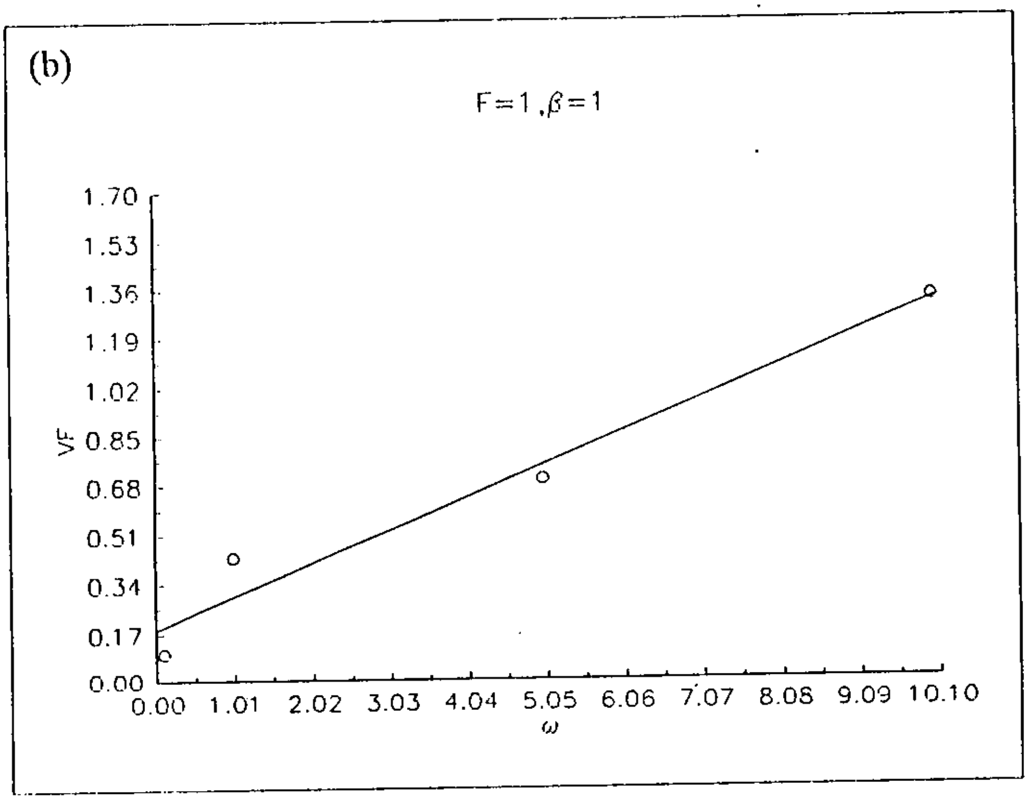


Figure (3.29) Effect of the amplitude of the exciting force on the vibration frequency


(a)
F=1,β=0.1
1.50
1.35
1.20
1.05
0.90
VF 0.75
0.60
0.45
0.30
0.15
0.00
0.00 1.01 2.02 3.03 4.04 5.05 6.06 7.07 8.08 9.09 10.10
ω


(b)
F=1,β=1
1.70
1.53
1.36
1.19
1.02
VF 0.85
0.68
0.51
0.34
0.17
0.00
0.00 1.01 2.02 3.03 4.04 5.05 6.06 7.07 8.08 9.09 10.10
ω

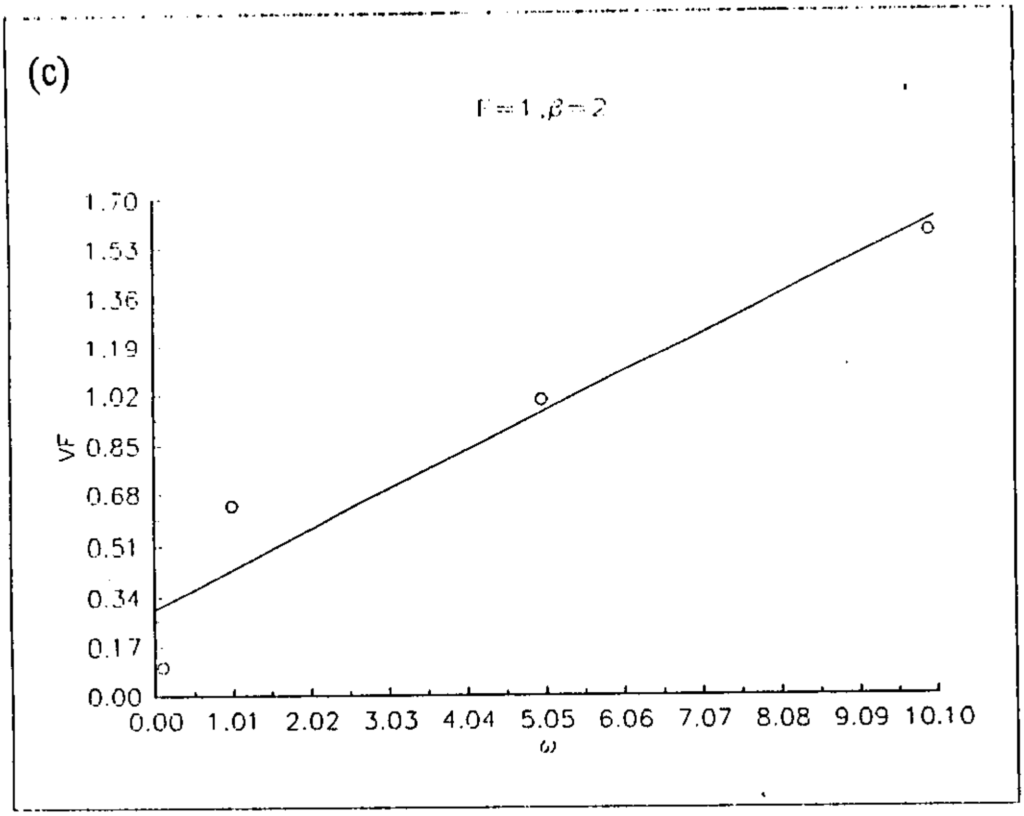


Figure (3.30) Effect of the frequency of the exciting force on the vibration frequency

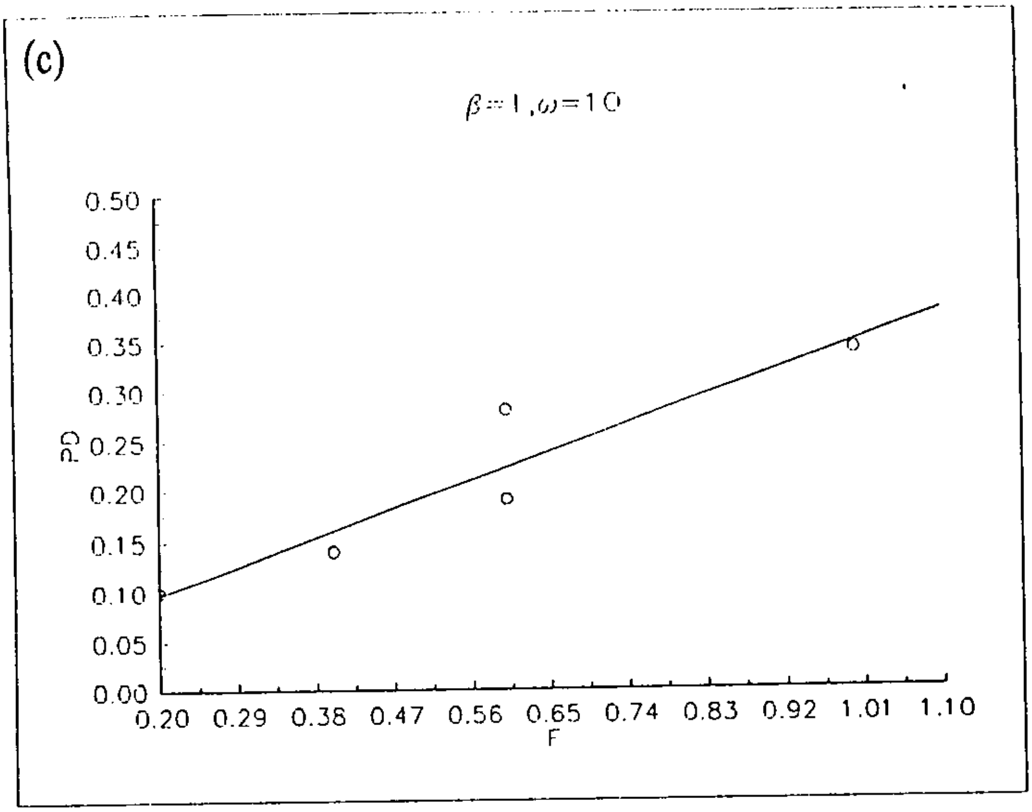
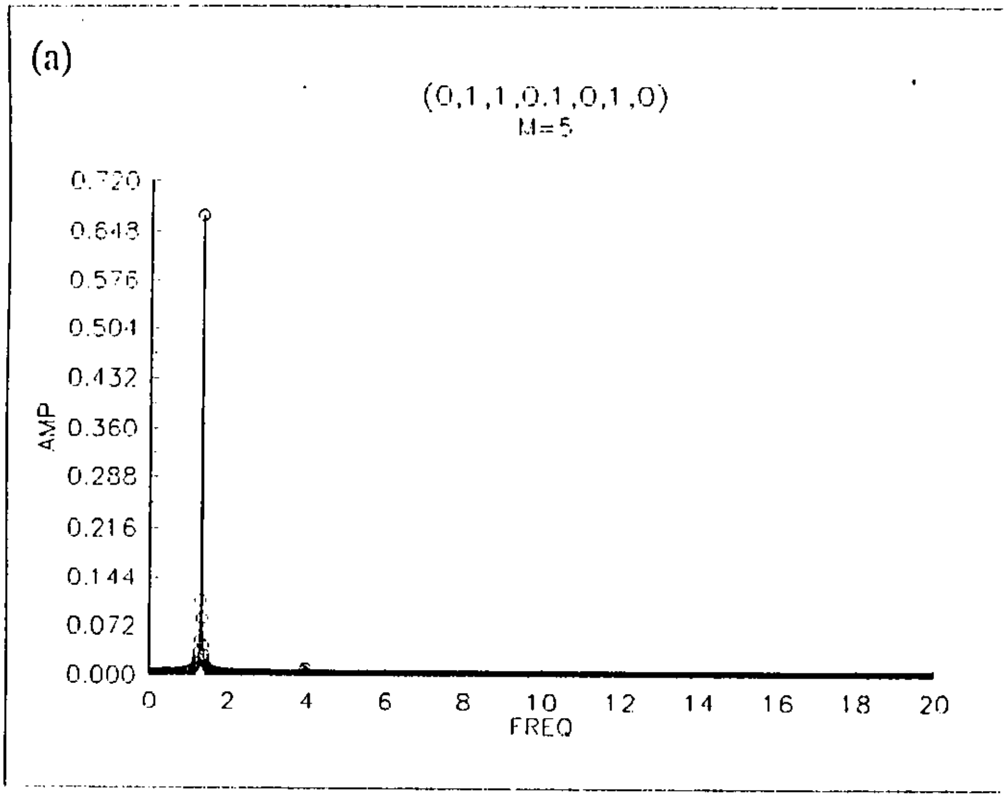


Figure (3.31) Effect of the nonlinear stiffness, the amplitude and frequency of the exciting force on the percentage deviation of the vibration frequency



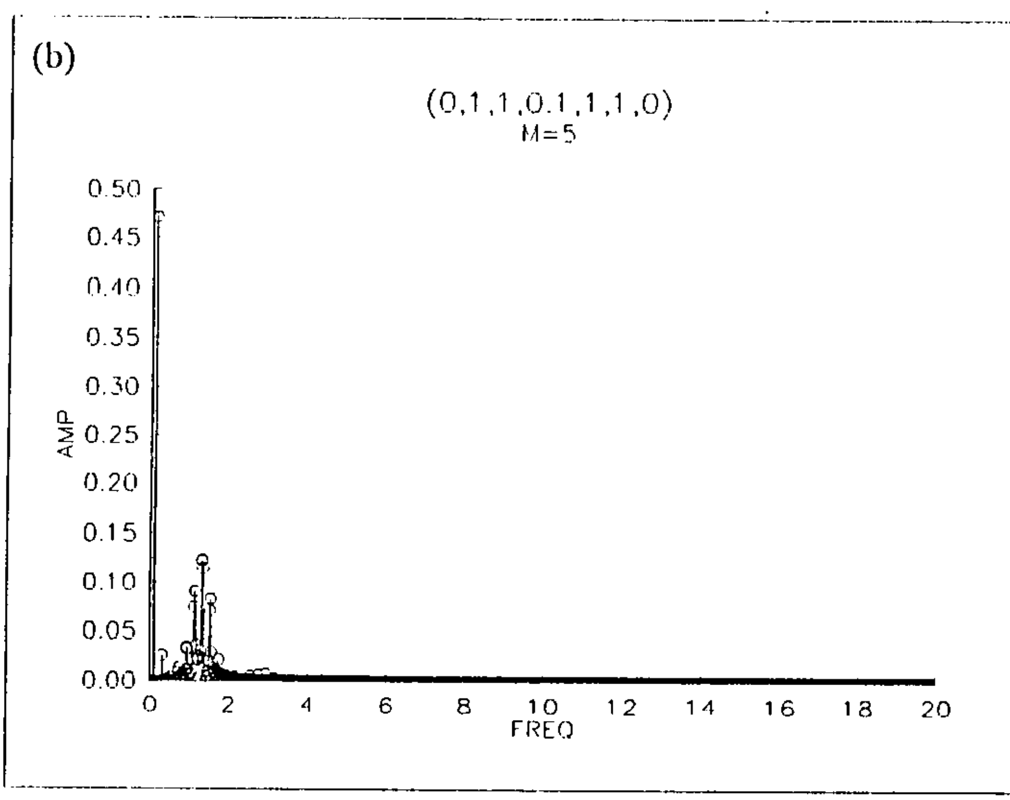
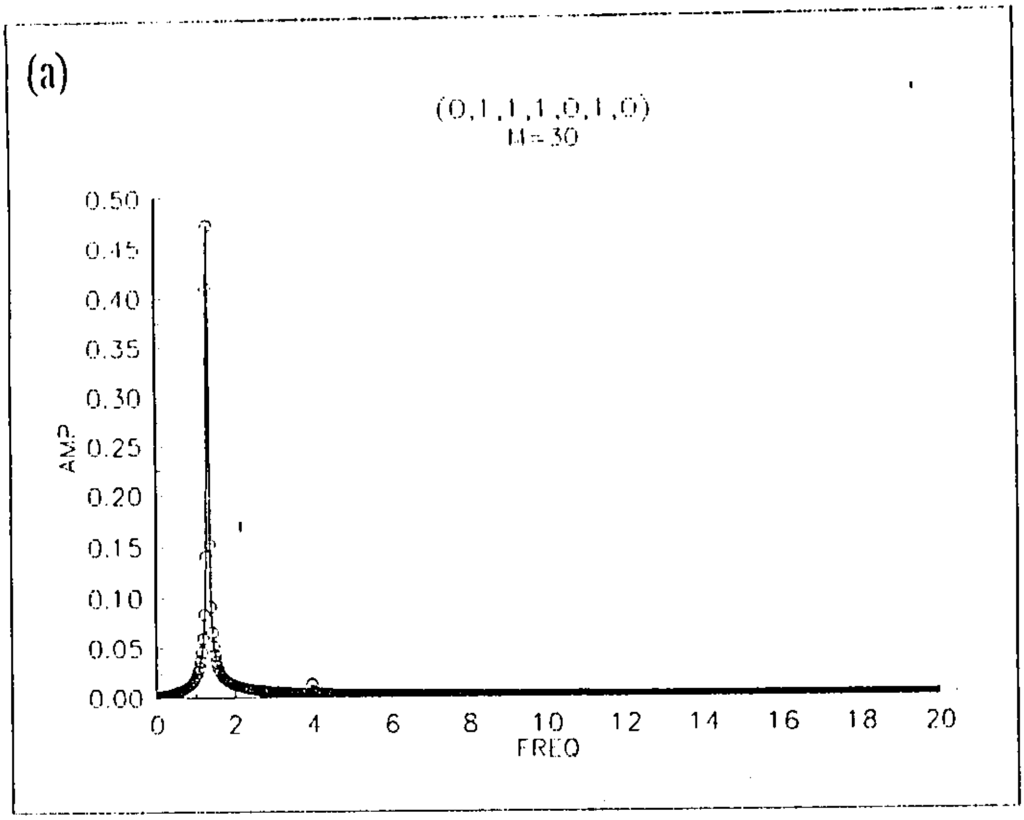


Figure (3.32) Frequency spectrum plots



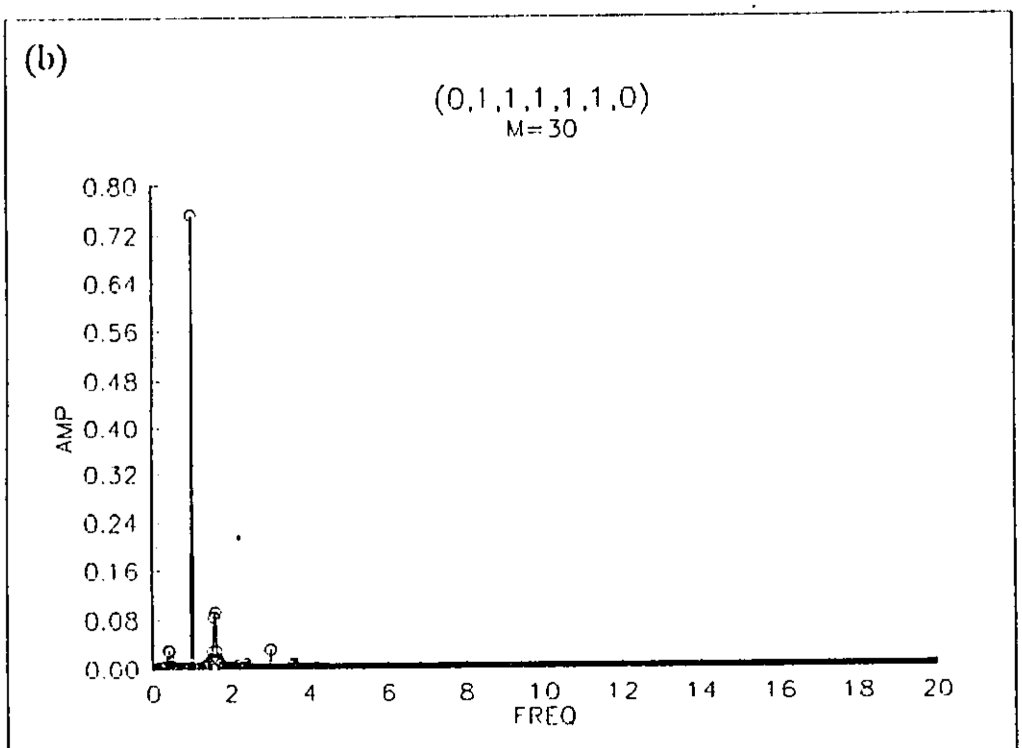


Figure (3.33) Frequency spectrum plots

CHAPTER FOUR

CONCLUSIONS

(1) For periodic response the waveform repeats itself at equal intervals of time, and the corresponding phase plane trajectory plots are fine closed orbits. For the non-periodic response the repetition of the waveform is not exactly constant but there is a small amount of shifting, the corresponding phase plane trajectory plots are not fine closed orbits but instead it spreads from the both sides or could take regular certain shapes.

(2) As the amplitude of the exciting force increases, the amplitude and the frequency modulation effect increase, and this effect is reduced as the exciting frequency becomes much larger than the free vibration frequency.

(3) The vibration frequency-time plot for periodic response was a horizontal straight line indicating a constant value. For the non- periodic response it is in general periodic when the forcing amplitude is not large. The changing nature of the vibration frequency becomes complicated for large forcing amplitude and the system undergoes chaotic motion.

(4) Increasing the nonlinear stiffness, increases the vibration frequency.

(5) Increasing the amplitude of the exciting force increases the vibration frequency.

(6) The relation between the frequency of the exciting force and the vibration frequency is direct, as the exciting frequency increases, the vibration frequency increases too.

(7) Increasing the nonlinear stiffness increases the percentage deviation of the vibration frequency.

(8) Increasing in the frequency of the exciting force will increase the percentage deviation at first until it reaches a maximum value (this value around the resonant case) then it begin to decrease. The vibration frequency resonates when the forcing frequency equals the free vibration frequency.

(9) Increasing the amplitude of the exciting force will increase the percentage deviation of the vibration frequency.

(10) The effect of the forcing frequency on the mean vibration frequency and on its percentage deviation are much greater than those of the forcing amplitude and the nonlinear stiffness.

(11) The error in the fourth order Runge-kutta method is of order h^5 and hence excellent accuracy was obtained by using this method, also it is stable and easy to program.

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APPENDICES

Appendix A : Runge-Kutta Methods

Each Runge-kutta method is derived from an appropriate Taylor method in such a way that the global truncation error is of order h^N. A trade-off is made to perform several function evaluations at each step and eliminate the necessity to compute the higher derivatives. These methods can be constructed for any order N. The Runge-Kutta method of order N=4 is most popular. It is a good choice for common purposes because it is quite accurate, stable, and easy to program. Most authorities proclaim that it is not necessary to go to a higher-order method because the increased accuracy is offset by additional computational effort. If more accuracy is required, then either a smaller step size or an adaptive method should be used.

The fourth-order Runge-Kutta method simulates the accuracy of the Taylor series method of order 4. The proof is algebraically complicated and results in a formula involving a linear combination of function values. The coefficients involved are chosen so that the method has a local truncation error of order h^5, and hence a global truncation error of order h^4.

Let $f(t, y)$ be a continuous function of t and y. The initial value problem is to solve

$$\frac{dy}{dt} = f(t, y) \quad \text{with} \quad y(t_0) = y_0$$

..........(A.1)

A solution to the initial value problem is a differential function y(t) with property that when t and y(t) are substituted in $f(t, y)$ the result is equal to the derivative $y'(t)$, that is

$$y'(t) = f(t, y(t)) \quad \text{and} \quad y(t_0) = y_0$$

..........(A.2)

The Runge-Kutta method requires that the interval is divided into subintervals of equal width h. Starting with (t_0, y_0), four function evaluations per step are required to generate the discrete approximations (t_i, y_i) as follows:

$$y_{i+1} = y_i + \frac{h}{6}(f_1 + 2f_2 + 2f_3 + f_4)$$

..........(A.3)

Where

$$f_1 = f(t_i, y_i)$$

$$f_2 = f(t_i + \frac{h}{2}, y_i + \frac{h}{2} f_1)$$

$$f_3 = f(t_i + \frac{h}{2}, y_i + \frac{h}{2} f_2)$$

$$f_4 = f(t_i + h, y_i + hf_3)$$

..........(A.4)

Appendix B : Fourier Transforms

The Fourier transform has become the underlying operation for the modern time series analysis. In many of the modern instruments for spectral analysis, the calculation performed is that of determining the amplitude and phase of a given record.

The Fourier integral is defined by the equation

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{i2\pi ft} df$$

..........(B.1)

In contrast to the summation of the discrete spectrum of sinusoids in the Fourier series, the Fourier integral may be regarded as a summation of the continuous spectrum of sinusoids. The quantity X(f) in the above equation is called the Fourier transform of x(t), which may be evaluated from the equation

$$X(f)=\int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}dt$$

..........(B.2)

Equation (B.2) resolves the function x(t) into harmonic component X(f), whereas Equation (B.1) synthesizes these harmonic components to the original time function x(t). The two equation above are referred to as the Fourier transform pair.

Fourier transform pair could apply to discrete time functions, that is, functions that are represented by a time series, a sequence of values at a discrete equispaced points which may come from experimental data or digital calculations.

The Fourier transform, Equation (B.1) and (B.2), for discrete time functions is computed by numerical integration, as follows

$$x(t_n)=\frac{1}{f_s}\int_{-f_s/2}^{f_s/2} X(f)e^{i2\pi ft_n}df$$

..........(B.3)

$$X(f)=\sum_{n=-\infty}^{\infty} x(t_n)e^{-i2\pi ft_n}$$

..........(B.4)

Where

$f_s=1/\Delta t$, is the sampling frequency.

$t_n=N\ \Delta t$, is the time corresponding to the nth time value.

N is the number of the data points.

If truncation is performed in the time series and discretization and truncation is similarly performed in the frequency function, the discrete transform becomes

$$x_n = \sum_{k=0}^{N-1} X_k e^{i(2\pi kn/N)} \qquad \text{..........(B.5)}$$

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-i(2\pi kn/N)} \qquad \text{..........(B.6)}$$

Because the infinite continuous integrals of Equations (B.1) and (B.2) have been replaced by finite summations, the transform pair above, known as the discrete Fourier transform (DFT), is much better adapted to digital calculations. Even so, it can be seen that in order to obtain N frequency components from N time points (or vice versa) requires N^2 complex multiplicati. A calculation procedure known as the fast Fourier transform (FFT) algorithm obtains the same result with much smaller number of complex multiplications.

Appendix C : Computer Programs:

Computer Program # 1:

```
C----------------------------------------------------------------------------------
C THIS COMPUTER PROGRAM IS USED TO FIND THE NUMERICAL
C SOLUTION OF THE FORCED DUFFING'S EQUATION USING THE
C FOURTH ORDER RUNGE-KUTTA METHOD, AFTER PLOTTING IT
C GIVES THE TIME HISTORY AND PHASE PLANE DIAGRAMS.
C----------------------------------------------------------------------------------

    DELTA=0.0
    ALPHA=1.0
    X0=1.0
    Y0=0.0
    READ(*,10)BETA,W,F,P
10  FORMAT(F10.6)

    PERIODF=6.283185/W
    H=PERIODF/400.0
    TF=P*PERIODF

    DO 20 T=H,TF,H
    F1=Y0
    G1=F*COS(W*T)-DELTA*Y0-ALPHA*X0-BETA*X0**3
    F2=Y0+H/2.*G1
   G2=F*COS(W*(T+H/2.))-DELTA*(Y0+H/2.*G1)-ALPHA*(X0+H/2.*F1)-
   .BETA*(X0+H/2.*F1)**3
    F3=Y0+H/2.*G2
   G3=F*COS(W*(T+H/2.))-DELTA*(Y0+H/2.*G2)-ALPHA*(X0+H/2.*F2)-
   .BETA*(X0+H/2.*F2)**3
    F4=Y0+H*G3
    G4=F*COS(W*(T+H))-DELTA*(Y0+H*G3)-ALPHA*(X0+H*F3)-
   .BETA*(X0+H*F3)**3
    X=X0+H/6.*(F1+2.*F2+2.*F3+F4)
    Y=Y0+H/6.*(G1+2.*G2+2.*G3+G4)
    WRITE(*,30)T,X,Y
30  FORMAT(1X,F8.4,1X,F14.6,1X,F14.6)
    X0=X
    Y0=Y
20  CONTINUE
    STOP
    END
```

Computer Program # 2:

```
C----------------------------------------------------------------------------------------
C THIS COMPUTER PROGRAM IS USED TO FIND THE VIBRATION
C FREQUENCY OF THE SYSTEM CORRESPONDING TO A CERTAIN
C WAVEFORMS WITH ANY EXPECTED SHAPE, AFTER PLOTTING IT
C GIVES THE VIBRATION FREQUENCY(VF)-TIME PLOTS.
C----------------------------------------------------------------------------------------

      REAL XX(800),XMAX(400),TMAX(400)
      INTEGER TYPE

      DELTA=0.0
      ALPHA=1.0
      X0=1.0
      Y0=0.0
      READ(*,10)BETA,W,F,P
   10 FORMAT(F10.6)

      PERIODF=6.283185/W
      H=PERIODF/400.
      TF=P*PERIODF

      I=0
      M=0
      XM=3.7
      TM=0.0
      READ(*,20)TYPE
   20 FORMAT(I1)

      DO 30 T=H,TF,H
      F1=Y0
      G1=F*COS(W*T)-DELTA*Y0-ALPHA*X0-BETA*X0**3
      F2=Y0+H/2.*G1
     G2=F*COS(W*(T+H/2.))-DELTA*(Y0+H/2.*G1)-ALPHA*(X0+H/2.*F1)-
     .BETA*(X0+H/2.*F1)**3
      F3=Y0+H/2.*G2
     G3=F*COS(W*(T+H/2.))-DELTA*(Y0+H/2.*G2)-ALPHA*(X0+H/2.*F2)-
     .BETA*(X0+H/2.*F2)**3
      F4=Y0+H*G3
      G4=F*COS(W*(T+H))-DELTA*(Y0+H*G3)-ALPHA*(X0+H*F3)
     .BETA*(X0+H*F3)**3
      X=X0+H/6.*(F1+2.*F2+2.*F3+F4)
      Y=Y0+H/6.*(G1+2.*G2+2.*G3+G4)
```

```
      IF(TYPE.EQ.1)THEN
            IF((X0*X).LT.0.)THEN
            I=I+1
            XX(I)=T-H/(X-X0)*X
            ENDIF
      ELSE
            IF(X.LE.0.0)GOTO 40
                  IF(X.GT.X0)THEN
                  TM=T
                  XM=X
                  ELSE
                        IF(X0.EQ.XM)THEN
                        I=I+1
                        TMAX(I)=TM
                        XMAX(I)=XM
                        ENDIF
                  ENDIF
            ENDIF
      ENDIF

40    X0=X
      Y0=Y
30    CONTINUE

      IF(TYPE.EQ.1)THEN
      DO 50 J=1,(I-1)/2
      TT=(XX(2*J+1)+XX(2*J-1))/2.
      WW=6.283185/(XX(2*J+1)-XX(2*J-1))
      WRITE(*,60)TT,WW
60    FORMAT(1X,F14.6,1X,F14.6)
50    CONTINUE
      ELSE
      XM=3.7
      TM=0.0
      DO 70 J=1,I-1
          IF(XMAX(J+1).GT.XMAX(J))THEN
          TM=TMAX(J+1)
          XM=XMAX(J+1)
          ELSE
              IF(XMAX(J).EQ.XM)THEN
              M=M+1
              XX(M)=TM
              ENDIF
          ENDIF
70    CONTINUE
```

```
      DO 80 L=1,M-1
      TT=(XX(L+1)+XX(L))/2.
      WW=6.283185/(XX(L+1)-XX(L))
      WRITE(*,90)TT,WW
90    FORMAT(1X,F14.6,1X,F14.6)
80    CONTINUE
      ENDIF

      STOP
      END
```

Computer Program # 3:

```
C-----------------------------------------------------------------------------------
C   THIS COMPUTER PROGRAM IS OBTAINED FROM THE NAG
C   LIBRARY WHICH IS USED TO FIND THE FREQUENCY
C   SPECTRUM PLOTS, BY USING THE (FFT) ALGORITHM.
C-----------------------------------------------------------------------------------

*     C06EAF Example Program Text
*     Mark 14 Revised. NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NMAX
      PARAMETER        (NMAX=4000)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      INTEGER          IFAIL, J, N, N2, NJ
*     .. Local Arrays ..
      DOUBLE PRECISION A(0:NMAX-1), B(0:NMAX-1), X(0:NMAX-1),
     +                 XX(0:NMAX-1),AMP(0:NMAX-1),POWER(0:NMAX-1)
*     .. External Subroutines ..
      EXTERNAL         C06EAF, C06EBF, C06GBF
*     .. Intrinsic Functions ..
      INTRINSIC        MOD
*     .. Executable Statements ..
      WRITE (NOUT,*) 'C06EAF Example Program Results'
*     Skip heading in data file

      SR=1/H
```

```
      READ (NIN,*)
      OPEN(NIN,FILE='DATA.OUT',STATUS='OLD')
20    READ (NIN,*,END=120) N
      IF (N.GT.1 .AND. N.LE.NMAX) THEN
         DO 40 J = 0, N - 1
         READ(NIN,*)X(J)
40       CONTINUE
         IFAIL = 0
*
      CALL C06EAF(X,N,IFAIL)
*
      WRITE (NOUT,*)
      WRITE (NOUT,*) 'Components of discrete Fourier transform'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '        Real      Imag'
      WRITE (NOUT,989)
989   FORMAT(2X,'N',6X,'Frequency(f)',3X,'Amplitude(C)',
     *3x,'Power(C^2/2)'/1X,'----',4X,'------------',3X,
     *'------------',3X,'------------')
         A(0) = X(0)
         B(0) = 0.0D0
         N2 = (N-1)/2
         DO 60 J = 1, N2
           NJ = N - J
           A(J) = X(J)
           A(NJ) = X(J)
           B(J) = X(NJ)
           B(NJ) = -X(NJ)
60       CONTINUE
         IF (MOD(N,2).EQ.0) THEN
           A(N2+1) = X(N2+1)
           B(N2+1) = 0.0D0
         END IF
         DO 80 J = 0, N - 1
           A(J)=A(J)/N**.5
           B(J)=B(J)/N**.5
           POWER(J)=(A(J)**2+B(J)**2)
           AMP(J)=SQRT(2.*POWER(J))
           WRITE (NOUT,99999) J, A(J), B(J)
80         CONTINUE
*
      CALL C06GBF(X,N,IFAIL)
      CALL C06EBF(X,N,IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,*)
```

```
      WRITE (NOUT,*)
   +  'Original sequence as restored by inverse transform'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '      Original  Restored'
      WRITE (NOUT,*)
      DO 100 J = 0, N - 1
        RESOL=SR/FLOAT(N)
        FREQ=J*RESOL
        WRITE (NOUT,99) J,FREQ, AMP(J),POWER(J)
99999   FORMAT(1X,I5,3F10.6)
100   CONTINUE
      GO TO 20
    ELSE
      WRITE (NOUT,*) 'Invalid value of N'
    END IF
120 STOP

99  FORMAT(I4,3(5X,F10.6))
    END
```

Appendix D: Amplitude & Frequency Modulations

The amplitude modulation is the process of varying the amplitude of the high frequency sinusoidal signal known as the carrier signal by a modulating signal. The resulting amplitude modulated waveform contains the carrier frequency, an upper side frequency equal to the summation of the carrier frequency and the modulation frequency, and a lower side frequency equal to their difference.

The frequency modulation is the process of varying the frequency of a carrier signal in proportion to a modulation signal. The carrier amplitude of a frequency modulated wave is kept constant during modulation.

ملخص باللغة العربية

دراسة عددية للاستجابة غير الدورية لمتذبذب الدفنغ المتعرض لقوة

456120

بدر الدين ماجد حسين بدر

إشراف

الدكتور مازن القيسي

في هذا العمل تم حل معادلة دفنغ القسرية التي تمثل معادلة الحركة لمتذبذب الدفنغ المتعرض لقوة عددياً باستخدام طريقة رنج-كتا من الدرجة الرابعة. تم دراسة الحل بمساعدة الماضي الزمني و رسومات المسار في مستوى الحالة بمدى متعدد من عوامل النظام. رسومات التردد الطيفي تم رسمها لمعرفة المحتوى الترددي للاشكال الموجية عند حالات معينة. بشكل عام, الاستجابة تكون غير دورية و الاستجابة الدورية تكون حالة خاصة منها.

لقد تم تصنيف نوعية الحركة الناتجة عن مختلف عوامل النظام الى دورية, و غير دورية. تم إيجاد تردد الاهتزاز و حساب الانحراف النسبي له. تم دراسة تأثير تغيير عوامل النظام على نوعية الحركة و على تردد الاهتزاز للنظام.

من هذه الدراسة وجد أنه عندما تكون الاستجابة دورية فأن شكل الموجة تكرر نفسها عند فترات متساوية من الزمن, و تكون رسومات المسار في مستوى الحالة مدارات مغلقة ناعمة. في حالة الاستجابة غير الدورية يكون التكرار لشكل الموجة ليس ثابتاً تماماً و لكن تكون هناك كمية قليلة من الازاحة, و تكون رسومات المسار في مستوى الحالة مداراتر مغلقة غير ناعمة و لكنها في المقابل تنتشر من كلا الجانبين أو تاخذ أشكال معينة منتظمة. كم وجد أنه كلما زادت قيمة الذروة للقوة المثيرة فان تاثير التغير في التردد و قيمة الذروة يزداد, هذا التغير يقل كلما أصبح التردد المثير أكبر بكثير من تردد الأهتزاز الحر.

وجد أيضاً أن شكل التردد الاهتزازي الناتج عندما تكون الاستجابة دورية كان خطاً مستقيماً أفقياً مشيراً لقيمة ثابتة, في حالة الاستجابة غير الدورية كان متقلباً حول قيمة وسطيةو بشكل عام كان دورياً. كما وجد أن الزيادة في التثبيت غير الخطي يزيد تردد الاهتزاز. إن الزيادة في قيمة الذروة و التردد للقوة المثيرة تزيد تردد الاهتزاز. وجد أن الانحراف النسبي لتردد الاهتزاز يزداد بالزيادة في التثبيت غير الخطي و الزيادة في قيمة ذروة القوة المثيرة. ظروف الرنين لتردد الاهتزاز بانحراف نسبي أقصى وجد أنه حصل عندما كان التردد المثير قريب لتردد الاهتزاز الحر.